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EEV, MCEV, Solvency, IFRS
a chance for actuarial mathematics to get to main-stream of insurance value chain

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Audit . Tax . Consulting . Financial Advisory.

Presentation outline

- **Current environment**
- **EEV, MCEV, Solvency**
- **Replicating portfolios**
- **Risk geographies**

Current environment

Increasing number of companies report:

EV Embedded Value – the value of adjusted net assets plus discounted value (RDR) of future profits from existing business minus the cost of capital

EEV European Embedded Value – the value of adjusted net assets plus discounted value (RDR) of future profits from existing business minus cost of financial options and guarantees minus the cost of capital

MCEV Market Consistent Embedded Value – the value where assets and liabilities are valued stochastically and where economic assumptions allow for replicating market value of assets

Current environment

- A valuation model is said to be ‘Market Consistent’ when it produces prices close to prices observed on financial markets for a number of reference assets
  - In this presentation ‘Fair Value’ ~ Market Consistent

- Market Consistent valuation of insurance liabilities (both capital and regulatory reserves) is now becoming the norm:
  - Convergence towards Market Consistent valuations (MCEV)
  - IFRS Phase II
  - Solvency II & Swiss Solvency Test
  - New product development (e.g. Variable Annuities)

- Ever more sophisticated stochastic financial simulation models

Market consistent valuations

Proper modelling demands complex sets of assumptions:

- several currencies
- several asset classes
  - bonds
  - shares
  - property

Market consistent valuations

Two approaches to economic environment:

- „risk neutral”
  - cash flows are projected and discounted with risk free rate
  - all asset classes have expected return equal to risk free rate, volatilities differ
- „real world”
  - discounting with risk adjusted rates
  - returns reflect risk of asset classes (expected returns of risky assets above risk free rate)

Stochastic discounting

Let $(\Omega, \mathcal{F}, P)$ denote probability space and $(\mathcal{F}_t)_{t=0,1,\ldots,n}$ an increasing sequence of σ-fields.

Suppose that we have a sequence

$$\mathbf{X} = (X_0, X_1, \ldots, X_n),$$

of random variables, where X_t is $\mathcal{F}_t$ measurable, t = 1, 2,…, n.

$\mathbf{X}$ is a **random cash flow** with single payments X_t at time t.

Assume that $\mathbf{X} \in L^2_{n+1}(P)$ and $Q : L^2_{n+1}(P) \to \mathbf{R}$ is a positive, continuous, linear functional on $L^2_{n+1}(P)$.

Stochastic discounting

Theorem (Riesz representation theorem)

There exists $\varphi \in L^2_{n+1}(P)$ such that for all $\mathbf{X} \in L^2_{n+1}(P)$ we have

$$Q(\mathbf{X}) = E\left[\sum_{t=0}^{n} X_t \varphi_t\right].$$

The vector φ (and its single components φ_t) is called **deflator**.

A deflator φ_t transports cash amount at time *t* to value at time 0.

Deflator φ_t can be decomposed into its **span-deflators**:

$$Y_t = \frac{\varphi_t}{\varphi_{t-1}},\ t > 0$$

which transport cash amount at time *t* to value at time *t*.

Stochastic discounting

The price process for a random vector $\mathbf{X} \in L^2_{n+1}(P)$ can be defined as follows

$$Q_t[\mathbf{X}] = Q[X \mid \mathrm{F}_t] = \frac{1}{\varphi_t} E\left[\sum_{t=0}^{n} X_t \varphi_t \middle| \mathrm{F}_t\right]$$

for $t = 0, 1, \ldots, n$.

The deflated price process

$$\left(Q_t[\mathbf{X}] \cdot \varphi_t\right)_{t=0,\ldots,n}$$

forms an F_t -martingale under P.

Market consistent valuations

An important actuarial analyses:

- Analysis of change:
  - Change of $\mathrm{EV}_{t-1} \rightarrow \mathrm{EV}_t$
  - Change of $\mathrm{MCEV}_{t-1} \rightarrow \mathrm{MCEV}_t$
- Bridge analysis
  - Change of $\mathrm{EV}_t \rightarrow \mathrm{MCEV}_t$
- In both analyses we separate the sources producing differencies. The order and type of analysis is important.
- Management decisions are to be based on analysis of change.

Replicating portfolios

- Portfolio of assets for which market value closely approximates the market consistent value of a given life insurance liability portfolio:
  - under today's market conditions, and
  - under future market conditions (i.e. for all future time periods and for all simulations)

- Comprises either directly tradable assets or “notional” assets (e.g. certain long-term options) for which a market-consistent price can be calculated easily and with reasonable accuracy

Benefits of using replicating portfolios

- Mirror model of liabilities

- Simulating a Replicating Portfolio of assets only much quicker and easier than current techniques (i.e. simulating directly the liability portfolio using an ALM model with the full balance sheet)
  - Frequent updates on valuations, risk measures and economic capital, construction of hedge programs …
  - Allows a quicker exploitation of existing models & resources freed up for other areas

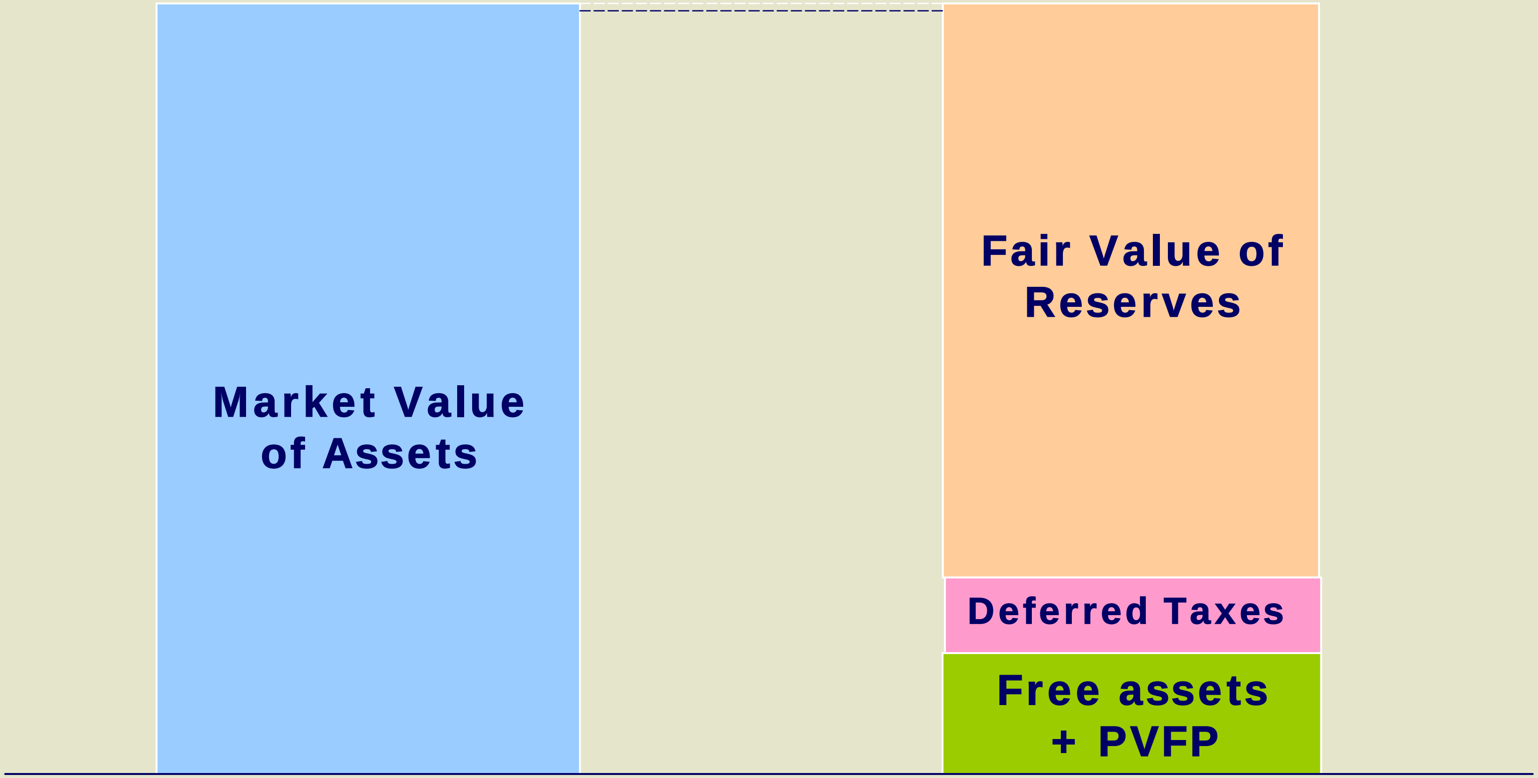
Market status

- Almost all pan-European insurers either already implemented or are seriously considering a move towards the use of Replicating Portfolio

- Rationale behind these projects:
  - Need for quicker economic capital calculations
  - Allocating capital appropriately across the group
  - Internal models for Solvency II / Swiss Solvency Test
  - Strategic asset allocation / Liability Driven Investment (LDI)
  - Quick and simple representation of the liabilities for use in financial analysis, risk limit setting and risk reporting

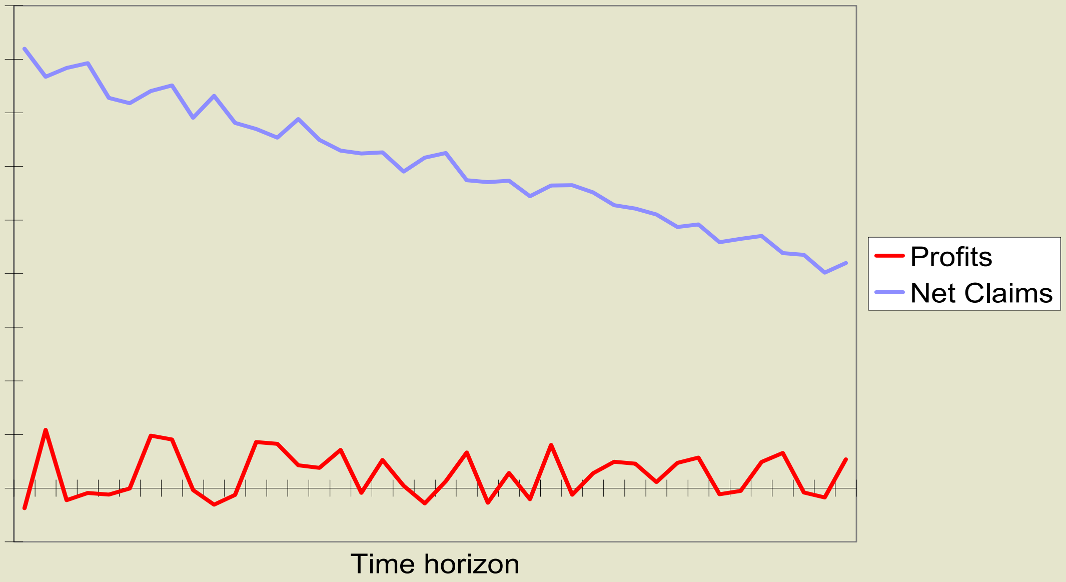
What is replicated: value vs. reserves

- Which part of the fair value balance sheet is replicated?



What is replicated: value vs. reserves

- According to aim different liabilities will be replicated:
  - MCEV focus: capital & present value of future profits
  - IFRS / reserves focus: present value of future net claims & costs
  - Economic capital focus: both
- Profits typically more volatile than claims



How to calculate replicating portfolios: cash-flows vs. balance sheets

- Two possible approaches:
  - Portfolio of assets whose future cash-flows match closely future liability cash flows in all scenarios
  - Portfolio of assets whose market value at t=0 matches directly the fair value balance sheet under various sensitivities at t=0
- Clear cost-benefit analysis in favor of cash-flows:
  - Replicating cash-flows requires 1 stochastic run and provides numerous conditions on the Replicating Portfolio of assets
  - Replicating directly fair value balance sheets requires several stochastic runs and provides only as many conditions

Optimal replicating portfolio

- Typical optimization program: finding weights wa for candidate assets that minimize (possibly under constraints):

$$\sum_{t \in times} \sum_{s \in simulations} \left[\left(\sum_{a \in assets} w_a CashFl._a(t,s) \right) - CashFl._{liab.}(t,s) \right]^2$$

  - Matching non discounted cash-flows → does not require Market Consistent economic scenarios
  - ≠ weightings may be applied for ≠ dates and ≠ scenarios
  - Other suitable norms than 2 (square) can be also used

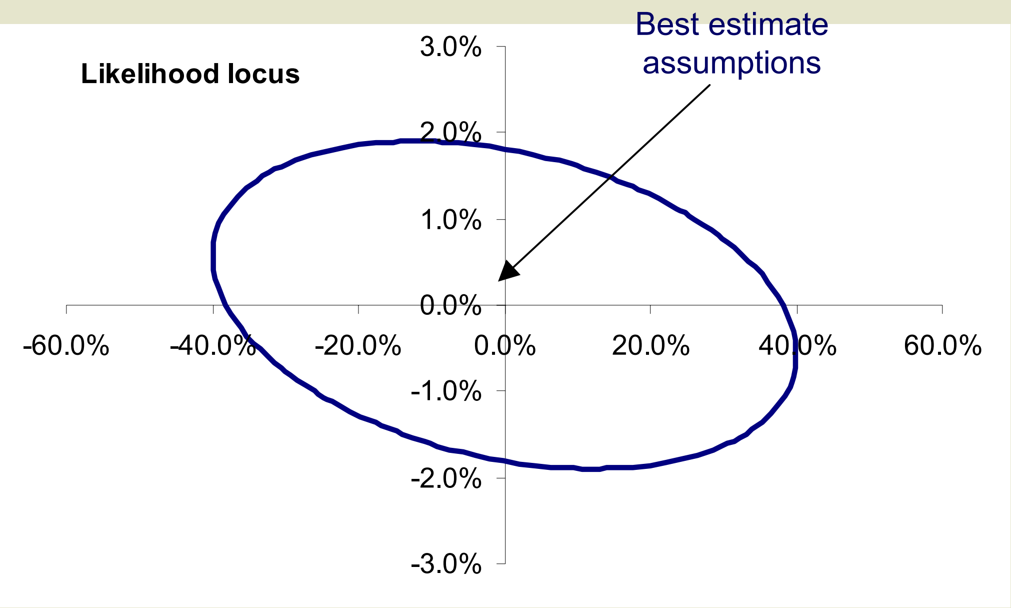
Applications – accelerated reporting

- Frequent (even daily) Fair Value reporting between two full-fledged calculations
  - Only need current market conditions to re-price Replicating Portfolio assets, not new set of scenarios

- Only about changes in financial markets conditions
  - Introduction of new business or policyholders' behavior deviations requires recalibration
  - If Replicating Portfolios are calculated by product / line of business then possible shortcuts with scaling factors

Applying risk geographies to solvency

- How do you identify an event at a given percentile for a multi-dimensional distribution?



Joint normal distribution is itself a normal distribution

From standard deviations and correlations we can derive overall standard deviation

Likelihood locus – points of equal probability density that are required number of standard deviations from best estimate

(i.e. for 99.5%ile, 2.58 sd's)

- Calculating ICA involves identifying **L**east **S**olvent **L**ikely **E**vent (LSLE)
- Also have the **M**ost **L**ikely **R**uin **E**vent (MLRE) but we focus on LSLE

Aggregation within ICA – Current Best Practice

Correlation Matrix

The majority of firms are using a correlation matrix approach as the primary means to calculate diversified capital requirements

Advantages
- Easy to implement
- Simple to communicate to senior management and the board
- Enables the ICA to be built up from first principles
- Makes it easier to target the required confidence interval

Disadvantages
- A lack of data to set individual correlation assumptions
- Assumes risks are Normally distributed
- Will not allow for non-linearity between risks
- May double count the impact of management actions

Scenario Testing – FSA “Medium bang” approach

Scenarios are calculated assuming a lower confidence interval for each risk so that when run simultaneously it represents a 1-in-200-year level event.

Advantages
- Should capture non-linearity impacts
- Will not double count the impact of management actions
- A subset of risks can be tested to target those areas where non-linearity is thought to exist

Disadvantages
- Difficult to assess the level of the reduced confidence interval
- Not clear that reducing the confidence of each individual stress by the same amount would generate the most onerous capital requirement at the required confidence interval

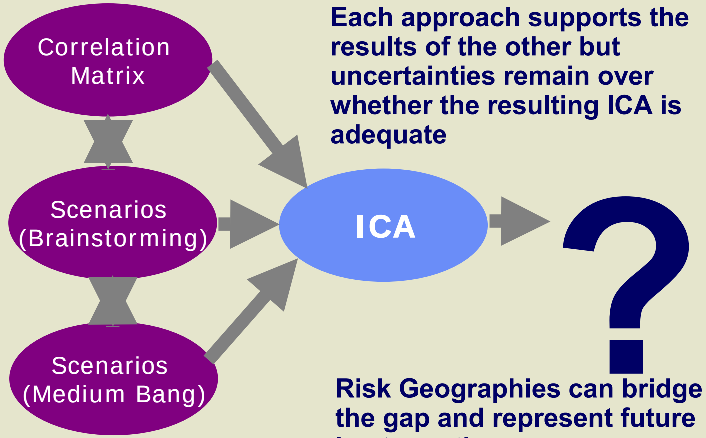
Scenario Testing – “Brainstorming” approach

Scenarios derived by considering possible adverse events and then brainstorming the knock-on effects within the business

Advantages
- A good way to engage other areas of the business, senior management and the board
- Helps to demonstrate the “Use test”
- Can help firms to assess the particular risks and combinations of those risks to which the company is exposed

Disadvantages
- Extremely difficult to target these scenarios at the required confidence interval
- Resulting capital requirements are typically considerably lower than the correlation matrix approach
- Very little reliance can be placed on the result

Introduces new techniques and extends analysis



Each approach supports the results of the other but uncertainties remain over whether the resulting ICA is adequate

Risk Geographies can bridge the gap and represent future best practice

Projecting fair values – Risk Geographies

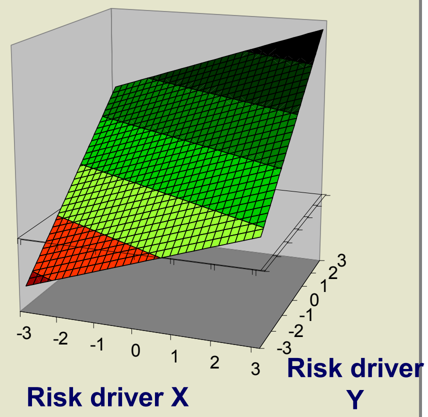
An Overview

- A best practice framework for economic capital calculation: can calculate the most onerous scenario at a given level of confidence
- The simplicity of the stress test and correlation approach
- The ease of communication of a scenario test
- Implicit allowance for non-linearity
- No new development of your models should be required for Risk Geographies

Response Function

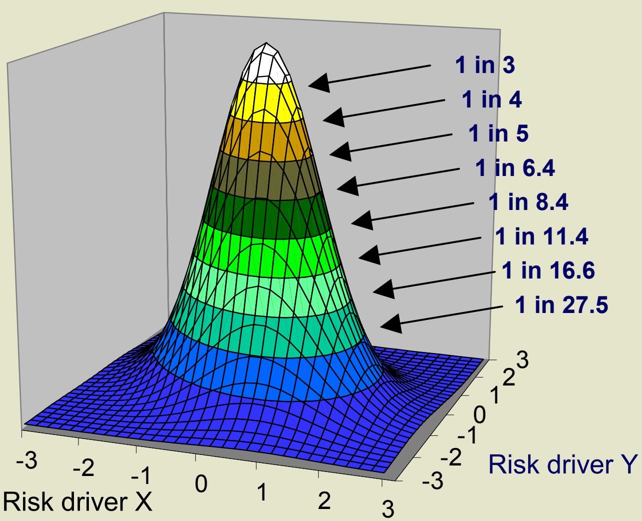
Consider a model firm whose net assets are exposed to two risk drivers – for example interest rates and stocks. In this example, we have standardised the risk drivers to have mean zero and standard deviation 1.

The response function expresses net assets as a function of risk drivers.

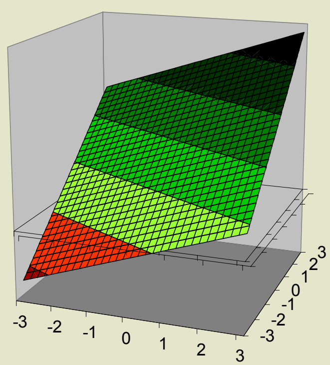


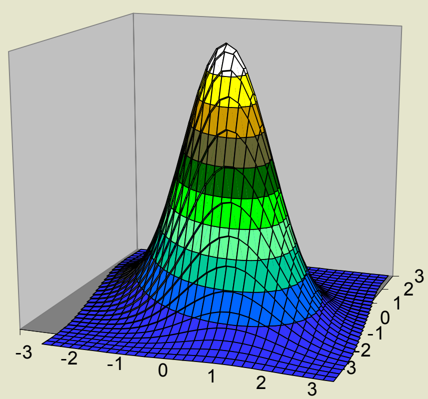
Likelihood Function

The likelihood function shows the probability density of various combinations of risk drivers. In this case, our likelihood function is a bivariate normal distribution with a characteristic bell-shape.



Where is likely and painful?

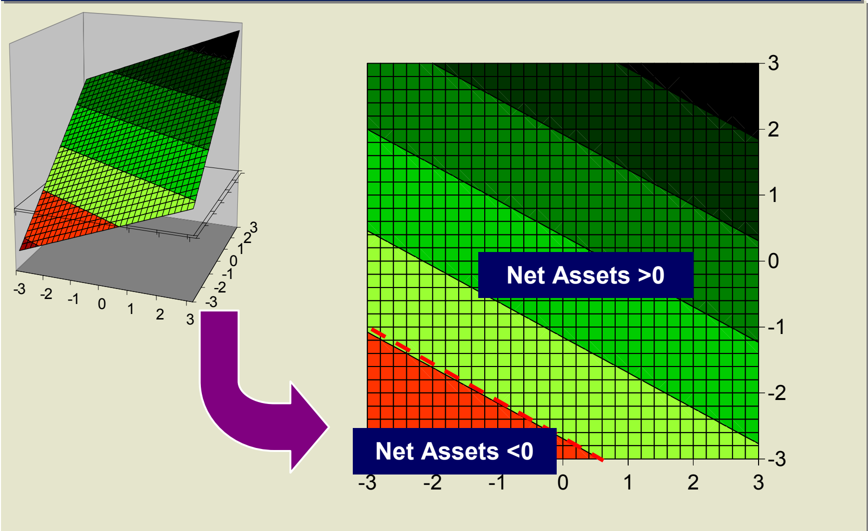




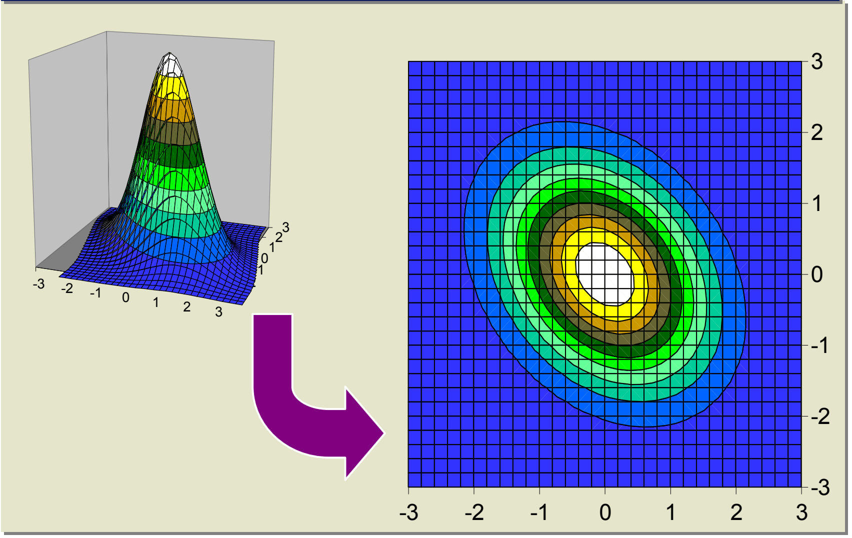
We seek to determine combinations of factor values which are simultaneously likely and painful.

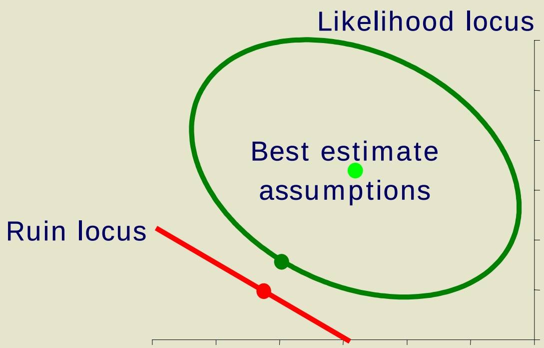
Projecting Fair Values – Risk Geographies

Birds-Eye View: Response

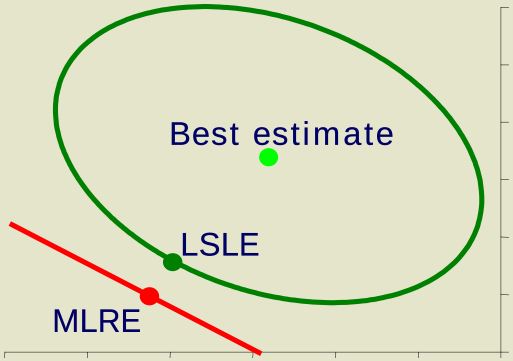


Birds-Eye View: Likelihood





Can you identify:
Most Likely Ruin Event (MLRE)?
Least Solvent Likely Event (LSLE)?



Applying Risk Geographies to ICA

How does it work?

- Risk geographies identifies the most onerous scenario at the required confidence level – the LSLE – the scenario that minimises net assets or solvency
- Iterative search process
- Should not require significant model changes
- Convergence generally takes place in 2-5 steps, depending on the number of factors involved and their non-linearity

Understanding and communicating risk

- Risk geographies highlights scenarios that drive capital requirements
- Can think through management response
    - Hedging
    - Reinsurance
    - Investment strategy
    - Contingent capital
- ...then calculate the LSLE and start again!

Summary

- A best practice framework for ICA
    - Can calculate the most onerous scenario at a given level of confidence
- The simplicity of the stress test and correlation approach
- The ease of communication of a scenario test
- Implicit allowance for non-linearity
- No new development of models should be required for Risk Geographies
    - If you can calculate an existing ICA you are equipped to perform the necessary calculations

Issues with valuation models

- Models require significant time & resources to be parameterized
  - Regular updates of stochastic valuations are difficult
- May take hours to produce today's fair value balance sheet
  - Calculation of Economic Capital (simulation of thousands of future fair value balance sheets) only with simplified models
- Hence models not really at the heart of the company's risk management & financial reporting
  - Key information only produced at great cost and distant intervals
  - Market moves: information quickly irrelevant
  - Local models not readily aggregated for group-level calculations

References

- Wüthrich M. V., Bühlmann H., Furrer H, „Market-Consistent Actuarial Valuation”, 2008
- Smith A. D., „Risk Geographies”, unpublished presentations

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