# Computable General Equlibrium

## The simple CGE model with production

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# Assumptions

- There is a single household with Cobb-Douglas preferences
- The household has an endowment of labour and capital
- Preferences are defined only over consumption goods
- Consumption goods are produced by the firms using Cobb-Douglas technology in a perfectly competitive setting
- Production of consumption goods uses capital and labour

# The SAM

| | X | Y | L | K | W | CONS | Total |
|---|---|---|---|---|---|---|---|
| X | | | | | 100 | | 100 |
| Y | | | | | 100 | | 100 |
| L | 40 | 60 | | | | | 100 |
| K | 60 | 40 | | | | | 100 |
| W | | | | | | 200 | 200 |
| CONS | | | 100 | 100 | | | 200 |
| Total | 100 | 100 | 100 | 100 | 200 | 200 | |

Use of factors in production
Consumers endowments/income
Consumption
Aggregate consumption

# Consumer side

Utility:

$$W = Ax_1^{\alpha_1} x_2^{\alpha_2}$$

$A$ will be called CONSCALE in our model, because we need to scale consumption in order that aggregate consumption = utility. From previous class we already know that:

$$x_1 = \alpha_1 \frac{INC}{p_1}$$

$$x_2 = \alpha_2 \frac{INC}{p_2}$$

# Income and aggregate utility

Income is derived from renting endowments of labour and capital to the firm:

$$INC = p_l \cdot \bar{L} + p_k \cdot \bar{K}$$

Agent does not save, so the value of aggregate utility has to equal to income:

$$p_w W = INC$$

And the price index of aggregate utility is equal to:

$$p_w = \frac{1}{A} \left( \frac{p_1}{\alpha} \right)^{\alpha} \left( \frac{p_2}{1-\alpha} \right)^{1-\alpha} = \frac{1}{A} \prod_{SEC} \left( \frac{p_{SEC}}{\alpha_{SEC}} \right)^{\alpha_{SEC}}$$

# Producer side

Production of X1:

$$xd_1 = B_1 K_1^{\beta_{K,1}} L_1^{\beta_{L,1}}$$

Production of X2:

$$xd_2 = B_2 K_2^{\beta_{K,2}} L_2^{\beta_{L,2}}$$

Demand for capital in sector SEC=1,2:

$$kd_{SEC} = \beta_{SEC} \frac{cost_{SEC}}{p_k}$$

Demand for labour in sector SEC=1,2:

$$ld_{SEC} = \beta_{SEC} \frac{cost_{SEC}}{p_l}$$

# Zero profits

Cobb douglas unit cost function:

$$cost_{SEC} = \frac{1}{B_{SEC}}\left(\frac{p_k}{\beta_{K,SEC}}\right)^{\beta_{K,SEC}}\left(\frac{p_l}{\beta_{L,SEC}}\right)^{\beta_{L,SEC}} = \frac{1}{B_{SEC}}\prod_{FAC}\left(\frac{p_{FAC}}{\beta_{SEC,FAC}}\right)^{\beta_{SE}}$$

We have zero profits, therefore:

$$p_{SEC} = \frac{1}{B_{SEC}}\prod_{FAC}\left(\frac{p_{FAC}}{\beta_{SEC,FAC}}\right)^{\beta_{SEC,FAC}}$$

# Model equations

- Zero profit on production of goods
- Determination of consumer price index
- Market clearing for aggregate consumption
- Market clearing for consumption goods
- Market clearing for factors (endowment = factor use)
- Budget constraint (determination of income)