Advanced Microeconomics: Homework 3

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Note: this is the only homework for the second part of the course. It also counts as more points.

1. There are two consumers A and B with the following utility functions and endowment

$$u_A(x_{1A}, x_{2A}) = x_{1A}x_{2A} \text{ and } \omega_A = (3, 2)$$

$$u_B(x_{1B}, x_{2B}) = \min(x_{1B}, x_{2B}) \text{ and } \omega_B = (2, 3).$$

   (a) Depict the situation in the diagram.

   (b) Calculate the market clearing prices and the equilibrium allocation.

2. Person A has a utility function of

$$u_A(x_{1A}, x_{2A}) = x_{1A} + 2x_{2A} \text{ and } \omega_A = (2, 0)$$

   Person B has a utility function of

$$u_B(x_{1B}, x_{2B}) = \min(2x_{1B}, x_{2B}) \text{ and } \omega_B = (0, 4).$$

   (a) Illustrate this situation in an Edgeworth box diagram.

   (b) What is the equilibrium relationship between p_1 and p_2.

   (c) What is the competitive equilibrium allocation?

3. Consider the Varian's revealed preference version of the second welfare theorem in the pure exchange economy. What assumption on preferences do you need to have $x_i' = x_i^*$ for all i. (In other words what assumptions do you need for the Walrasian equilibrium to guarantee that the competitive equilibrium will be exactly in the allocated Pareto efficient point).

4. Consider a production economy with one consumer with a Cobb-Douglas utility function for leisure x_1 and consumption x_2 : $u(x_1, x_2) = 0.5 \ln x_1 + 0.5 \ln x_2$. Endowment of leisure is one. Consumer can supply its leisure to the firm with technology $q = L - a$, where L is the supply of labour to the firm and $a \geq 0$ is a constant and q is the output of good 2. Consumer owns the firm.

   (a) Does the competitive equilibrium exist? Why? What is the condition for a for the competitive equilibrium to exist?

   (b) Find the Pareto optimal allocation.

   (c) Draw your findings on a diagram depicting the production set and the indifference curves.

5. Consider a production economy with one consumer with a Cobb-Douglas utility function for leisure $x_1$and consumption x_2 : $u(x_1, x_2) = x_1^{0.5}x_2^{0.5}$. Endowment of leisure is one. Consumer can supply its leisure to the firm with a techology $q = L$, where L is the supply of labour to the firm. Consumer owns the firm.

   (a) Draw the production set and the indifference curves in a single diagram.

   (b) Does the competitive equilibrium exist? Find the competitive allocation prices and mark your answer on the diagram.

   (c) Is the equilibrium (if it exists) Pareto optimal? Why?

6. Consider a production economy with one consumer with a utility function for leisure $x_1$and consumption x_2 : $u(x_1, x_2) = \min\{x_1, x_2\}$. Endowment of leisure is one. Consumer can supply its leisure to the firm with a technology $q = \sqrt{L}$, where L is the supply of labour to the firm. Consumer owns the firm.

   (a) Draw the production set and the indifference curves in a single diagram.

   (b) Does the competitive equilibrium exist? Find the competitive allocation prices and mark your answer on the diagram.

   (c) Is the equilibrium (if it exists) Pareto optimal? Why?

7. (Small open economy with a Leontief function) Assume that we have two countries, each producing two goods: crackers and beer. Technology of production is the same in the two countries:

$$Q_c = \min\{\frac{1}{2}K, 1L\} \text{ and} Q_b = \min\{\frac{1}{3}K, \frac{1}{4}L\}.$$

   Endowments are: at home: $\omega_L = \omega_K = 100$, abroad: $\omega_L^* = 120$, $\omega_K^* = 80$.

(a) Which of the goods is more labour intensive? What would be your expectations towards the production pattern i.e. who will produce more of what?

(b) Compute equilibrium production level assuming that there is full employment, i.e. all factors of production are used.

8. (Small open economy with a Cobb-Douglas function) Assume we have an economy consisting of two firms producing guns and oranges from capital and labour. Assume that we have a production function for oranges:

$$o = K^{1/3}L^{2/3}$$

and for guns:

$$g = K^{2/3}L^{1/3},$$

where K and L are capital and labour.
The associated cost functions are $C(g, w, r) = (\frac{w}{1/3})^{1/3}(\frac{r}{2/3})^{2/3}g$ and $C(o, w, r) = (\frac{w}{2/3})^{2/3}(\frac{r}{1/3})^{1/3}o$. Assume that this economy is a small open economy with endowments $\bar{K} = 1$ and $\bar{L} = 1$. The world prices are $p_o = 1$, $p_g = 1$. The producers trade directly with the rest of the world. Find the equilibrium in the goods and factor markets:

(a) Find the equilibrium factor wages. (Hint: use the equation 15.D.7 from the book).

(b) Derive the unit labour and capital demands a_{gL},a_{oL},a_{gK},a_{oK}.

(c) Find the factor allocation.

(d) Find the the output levels of final goods.

9. (2 firms x 2 consumers x 1 input economy) Consider an economy with two firms and two consumers. Firm 1 is entirely owned by consumer 1. It produces guns from oil via the production function $g = 2x$, where x is the input of oil. Firm 2 is entirely owned by consumer 2. It produces butter from oil via the production function $b = 3x$. Each consumer owns 10 units of oil. Consumer 1's utility function is $u_1(g_1, b_1) = g_1^{0.4}b_1^{0.6}$ and consumer 2's utility function is $u_2(g_2, b_2) = g_2^{0.5}b_2^{0.5}$.

(a) Find the market clearing prices for guns, butter and oil (note that consumers do not derive utility from consumption of oil alone). Use the price of oil as numeraire (set it equal to 1).

(b) How many guns and how much butter does each consumer consume?

(c) How much oil does each firm use?

10. (2 goods/firms x 2 inputs x 2 countries, dr Leszek Wincenciak) A closed economy (Home) produces two goods: Q_1 and Q_2 using capital K and labour L. The production function in the $i = 1, 2$-th sector is given by:

$$Q_i = (K_i)^{\alpha_i}(L_i)^{1-\alpha_i}$$

where $\alpha_1 = \frac{1}{3}$ and $\alpha_2 = \frac{2}{3}$. The preferences of a representative consumer is given by the following utility function:

$$U(x_1, x_2) = \beta \ln x_1 + (1 - \beta) \ln x_2$$

where $\beta = \frac{1}{2}$.

(a) Find optimal factor ratios (K/L) in both sectors as a function of wages.

(b) Find employment level L_1 in sector 1 as a function of labour and capital endowments and the ratio factor wages. Use the resource constraint.

(c) Find the final good price ratio P_1/P_2 as a function of factor wages.

(d) Write the production level of good 1 as a function of P_1/P_2 and factor endowments (use your previous findings from (a)-(c).

(e) Write down the consumer demand as a function of P_1 and P_2 and factor endowments.

(f) Using the market clearing condition for good 1 $(x_1 = Q_1)$ find the resulting price ratio P_1/P_2 as a function of factor endowments.

(g) If there is a second economy (Foreign, denoted by $*$) that differs only by endowments. When will our Home economy be an exporter of good 1 (eg. when good 1 will be relatively cheaper in Home than in Foreign)

11. Look for conditions of interior solution Pareto optimality (use the same assumptions as chapter 16.F in MWG), by:

(a) setting up and finding the first order conditions of the following problem using the given Lagrange multiplier (Pareto optimal solutions given initial level utility of consumers $i = 2, \ldots, I$):

$$\begin{array}{lll} Max & u_1(x_{11}, \ldots, x_{L1}) & & \\ \text{st} & u_i(x_{1i}, \ldots, x_{Li}) \geq \bar{u}_i & i = 2, \ldots, I & \text{multiplier } \delta_i \\ & \sum_i x_{\ell i} \leq \bar{\omega}_\ell + \sum_j y_{\ell j} & \ell = 1, \ldots, L & \text{multiplier } \mu_\ell \\ & F_j(y_{1j}, \ldots, y_{Lj}) \leq 0 & j = 1, \ldots, L & \text{multiplier } \gamma_j \end{array}$$

(b) deriving conditions:

$$\begin{array}{ll} \frac{\partial u_i/\partial x_{\ell i}}{\partial u_i/\partial x_{\ell' i}} = \frac{\partial u_{i'}/\partial x_{\ell i'}}{\partial u_{i'}/\partial x_{\ell' i'}} & \text{for all } i, i', \ell, \ell' \\ \frac{\partial F_j/\partial y_{\ell j}}{\partial F_j/\partial y_{\ell' j}} = \frac{\partial F_{j'}/\partial y_{\ell j'}}{\partial F_{j'}/\partial y_{\ell' j'}} & \text{for all } j, j', \ell, \ell' \\ \frac{\partial u_i/\partial x_{\ell i}}{\partial u_i/\partial x_{\ell' i}} = \frac{\partial F_j/\partial y_{\ell j}}{\partial F_j/\partial y_{\ell' j}} & \text{for all } i, j, \ell, \ell' \end{array}$$

(c) Explain in words, what the above conditions mean.

(d) Derive the above first order conditions from either:

i. finding the "social planner solution" as in the lecture slides (one maximization problem) or:

ii. finding the "competitive equilibrium solution" as in the lecture slides (two separate sets of problems - for the firms and for the consumers)