Homework 1

Let me know if you spot errors in the homework.

1. Let u be a utility function which generates demand function $x(p, w)$ and indirect utility function $v(p, w)$. Let be $F : \mathbb{R} \to \mathbb{R}$ a strictly increasing function. If the utility function u^* is defined by $u^*(x) = F(u(x))$ what are the demand functions generated by u^*(answer in terms of $x(p, w)$ and $v(p, w)$).

2. Let u be a utility function which generates Hicksian demand function $h(p, u)$ and expenditure function $e(p, u)$. Let be $F : \mathbb{R} \to \mathbb{R}$ a strictly increasing function. If the utility function u^* is defined by $u^*(x) = F(u(x))$ what are the Hicksian demand functions generated by u^*(answer in terms of $h(p, u)$ and $e(p, u)$). How are the expenditure functions related?

3. Rederive Cobb-Douglas Walrasian and Hicksian demands for the general case:

$$U(x) = \prod_{l=1}^{L} x_l^{\alpha_l}$$

   with $\sum_{l=1}^{N} \alpha_l = 1$.

4. For the general Cobb-Douglas utility function, derive the indirect utility function and the expenditure function.

5. For the utility function $u(x) = \sum_{l=1}^{L} \alpha_l \ln(x_l - \gamma_l)$, where $\sum_{l=1}^{N} \alpha_l = 1$ and $\gamma_l < 0$

   (a) find the demand function and indirect utility function for the case $l = 2$ (look for corner solutions).

   (b) use the provided GAMS code and check how changes in the γ_1 with $\alpha_1 = \alpha_2 = 0.5$ affect the income elasticities of demand.

6. For the Leontieff utility function $u(x) = \min\{x_1/a_1, x_2/a_2, \ldots, x_l/a_l\}$ where $a_l > 0, \forall_l$, find the demand function and the indirect utility function. What is the expenditure function?

7. For the utility function $u(x) = x_1 + \sqrt{x_2} + \sqrt{x_3}$ find the demand function and the indirect utility function. Beware: corner solutions! What kind of preferences such a function represents?

8. If a consumer has preferences that generate the indirect utility function $v(p, w) = w/(p_1 + p_2)$, what is the consumer expenditure function.

9. If a consumer has expenditure function $e(p, u) = au(p_1 b + p_2 c)$, where a, b, c are strictly positive constants, what is the Hicksian demand function for good 1.

10. If a consumer has an indirect utility function $v(p, w) = w[p_1^r + p_2^r]^{-1/r}$, derive the Walrasian demand function.

11. For a constant elasticity of substitution (CES) utlility function

$$u(x_1, x_2) = (x_1^\rho + x_2^\rho)^{1/\rho}.$$

   (a) Find Walrasian demand, Hicksian demand, indirect utility and expenditure functions

   (b) Find the income elasticity of demand and the own price elasticity of demand.

12. Consider a consumer with the following expenditure function:

$$e(p_1, p_2, u) = 2p_1^{1/2} p_2^{1/2} u^{1/2}$$

   and the initial prices of the two goods are: $p_1 = 2$ and $p_2 = 1$. Initially the consumer demands goods 1 and 2 and spends 40 PLN on the goods he consumes.

   (a) what is the consumer initial utility level?

(b) suppose that price p_1 falls to 1. To analyze a change in the consumer welfare measured in monetary terms, compute the compensating variation (CV).

(c) interpret your result (explain in words what the number obtained in b means for the consumer).

(d) suppose that you know the new utility level at prices $p_1 = 1$ and $p_2 = 1$ and that new utility is equal to 400. Compute the equivalent variation (EV).

(e) what does the relationship between the two measures imply for the type of the goods 1 and 2 (normal/inferior).

13. (Varian 10.2) Ellsworth's utility function is $U(x, y) = min\{x, y\}$. Ellsworth has 150\$ and the price of x and the price of y are both 1. His boss is about to send him to another town where the price of x is 1 and the price of y is 2. The boss offers no rise in pay. For Ellsworth, the move is exactly as bad as a cut in pay equal to A\$. He would not mind moving if he got a raise equal to B\$. Compute A and B.

14. When consumer income is 500, the demand functions are $x_1(p_1, p_2, 500) = \frac{200}{p_1}$, $x_2(p_1, p_2, 500) = \frac{300}{p_2}$. Can this information be used to determine which of the two situations the consumer prefers. Situation 1: $p_1 = 2$, $p_2 = 4$, $w = 200$ and situation 2: $p_1 = 3$, $p_2 = 15$, $w = 300$. Hint: use the fact that that consumer demand is homogeneous of degree zero in prices and income, so that if you double prices and income, consumer's situation does not change.

15. When consumer income is 500, the demand functions are $x_1(p_1, p_2, 500) = \frac{200}{p_1}$, $x_2(p_1, p_2, 500) = \frac{300}{p_2}$. Can this information be used to determine which of the two situations the consumer prefers. Situation 1: $p_1 = 2$, $p_2 = 4$, $w = 200$ and situation 2: $p_1 = 6$, $p_2 = 3$, $w = 300$. Hint: use the fact that that consumer demand is homogeneous of degree zero in prices and income, so that if you double prices and income, consumer's situation does not change.

16. (Varian 9.11) Consumer 1 has expenditure function $e_1(p_1, p_2, u_1) = u_1\sqrt{p_1 p_2}$ and consumer 2 has a utility function equal to $u_2(x_1, x_2) = 43x_1^3 x_2^a$.

(a) what are the Walrasian demand functions for each of the goods by each of the consumers?

(b) for what value(s) of the parameter a will there exist an aggregate demand function that is independent of the distribution of income (hint: paralell wealth expansion paths)?