Advanced Microeconomics

Pure exchange economies

Jan Hagemejer

November 24, 2013

Introduction

- We move on to **general equilibrium** where:
  - we put together demand and supply
  - all prices are variable (we model complete economies) and we take into account interactions between agents (unlike **partial equilibrium**)
- We begin by a situation where all economic agents are consumers (**pure exchange**).
- We will later move to a framework where agents own **factors of production** that they will rent to firms and at the same time buy **consumption goods** from firms.

Pure exchange economy: the setup

- A *pure exchange economy* is an economy in which there are no production opportunities.
- Consumer own initial stock or *endowments* of commodities.
- Economic activity is limited to trading and consumption.
- The simplest case involves only **two consumers**.

Pure exchange economy: the setup

- Consumers $i = 1, 2$.
- Commodities $\ell = 1, 2$.
- Consumer i consumption set is $\mathbb{R}^2_+$.
- Preference relation $\succeq_i$ over consumption vectors in this set.
- Endowment vector $\omega_i = (\omega_{1i}, \omega_{2i})$.
- Total endowment of good ℓ in the economy $\bar{\omega}_\ell = \omega_{\ell 1} + \omega_{\ell 2}$. Assume $\bar{\omega}_\ell > 0$.

Feasible allocations

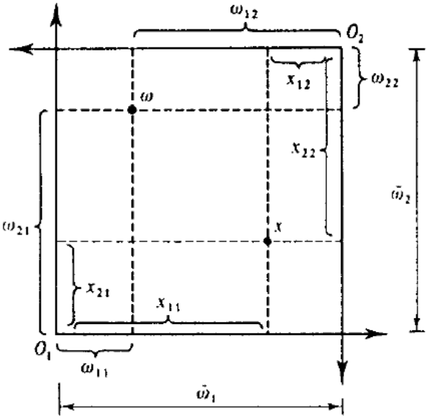
Definition

An ***allocation*** $x \in \mathbb{R}^4_+$ *in this economy is an assignment of a non-negative consumption vector to each consumer:*
$x = (x_1, x_2) = ((x_{11}, x_{21}), (x_{12}, x_{22}))$.
An allocation is ***feasible*** *for the economy if:*

$$x_{\ell 1} + x_{\ell 2} \leq \bar{\omega}_\ell \text{ for } \ell = 1, 2.$$

- An allocation for which the above holds with equality is **non-wasteful**.
- All non-wasteful allocations can be depicted in an Edgeworth box.

An Edgeworth box



- All the endowment of goods in the economy are consumed either by consumer 1 or 2
- Non-wasteful allocations: $(x_{12}, x_{22}) = (\bar{\omega}_1 - x_{11}, \bar{\omega}_2 - x_{21})$.

The budget set.

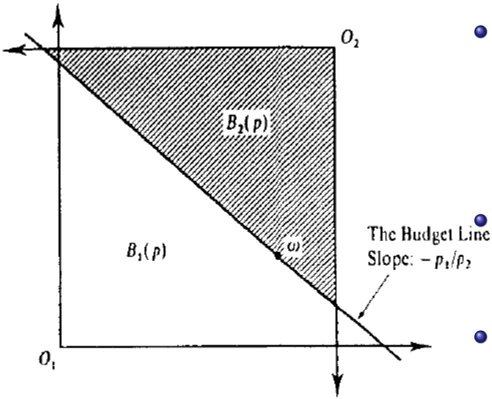
- The wealth of a consumer is not given exogeneously. It is determined by the level of prices.
- For any price $p = (p_1, p_2)$, the wealth equals the market value of his endowments of commodities:

$$p \cdot \omega_i = p_1\omega_{1i} + p_2\omega_{2i}.$$

- Given the endowment vector ω_i, the budget set of a consumer is solely a function of prices:

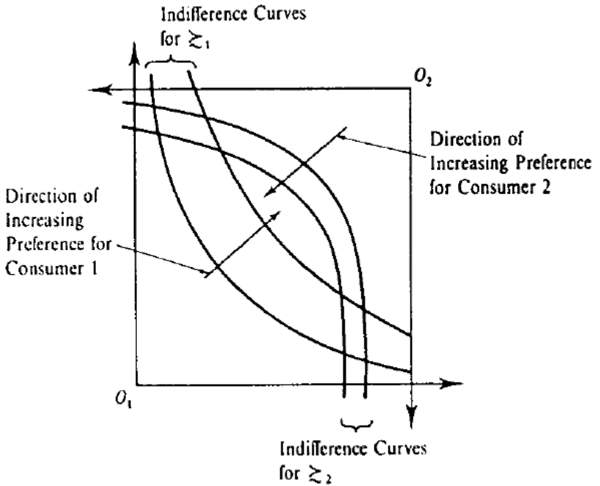
$$B_i(p) = x_i \in \mathbb{R}^2_+ : p \cdot x_i \leq p \cdot \omega_i.$$

The budget set.



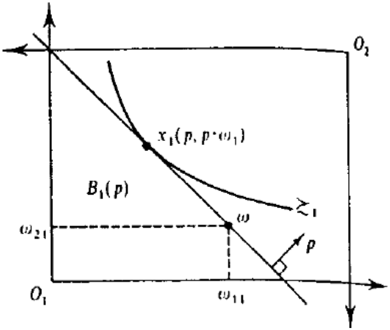
- The budget sets can be represented by a line through the endowment point with slope $-(p_1/p_2)$.
- All points below the budget line are affordable to consumer 1.
- All points above the budget line are affordable to consumer 2.
- The points on the budget line are available to both consumers at prices (p_1, p_2).

Preferences in the Edgeworth box



- Assume preferences are in general strictly convex, continuous, strongly monotone.

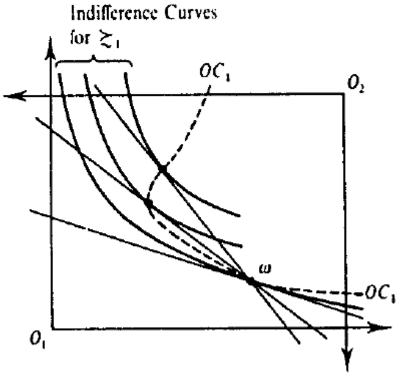
Optimal choice of a consumer

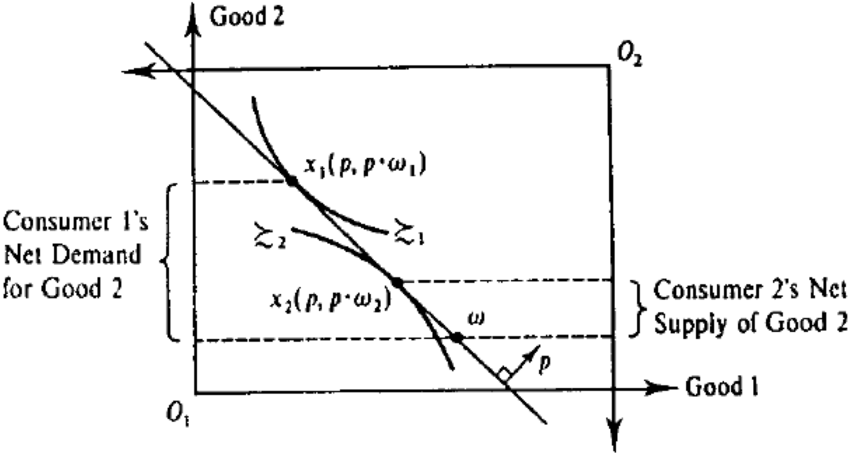


- Given p, the consumer 1 demands his most preferred point in $B_1(p)$ - demand function $x_1(p, p \cdot \omega_1)$.

Offer curve

- An offer curve traces all points (x_{1i}, x_{2i}) - the consumer i demands at each level of relative prices p_1, p_2.
- Tangency to the indifference curve at ω_1.




Good 2
O_2
$x_1(p, p\cdot\omega_1)$
Consumer 1's Net Demand for Good 2
$\succsim_2$
$\succsim_1$
$x_2(p, p\cdot\omega_2)$
ω
Consumer 2's Net Supply of Good 2
p
Good 1
O_1

Equilibrium

Definition

A *Walrasian (or competitive)* equilibrium for an Edgeworth box economy is a price vector $p*$ and an allocation $x^* = (x_1^*, x_2^*)$ in the Edgeworth box such that for $i = 1, 2,$:

$$x_i^* \succeq_i x_i' \text{ for all } x_i' \in B_i(p^*).$$

- In an equilibrium market clears:
  - note that by definition of E. box the allocation is non-wasteful which means:
  - excess demand equals 0. Demand equals supply (for all goods).
  - the offer curves of two consumers intersect.
- Important: each consumer's demand is homogeneous of degree zero in the price vector $p = (p_1, p_2)$ - if prices double, consumer's wealth doubles and the budget set remains unchanged. If $p*$ is an equilibrium price vector then any $\alpha p*$ is an equilibrium price vector for any $\alpha > 0$.

Equilibrium with utility functions

Definition

A *Walrasian (or competitive)* equilibrium for an Edgeworth box economy with consumers whose preferences are represented by the utility function $u_i(x_i)$ is a price vector p^* and an allocation $x^* = (x_1^*, x_2^*)$ in the Edgeworth box such that for $i = 1, 2,$:

$$u(x_i^*) \geq u_i(x_i') \text{ for all } x_i' \in B_i(p^*).$$

Equilibrium with utility functions

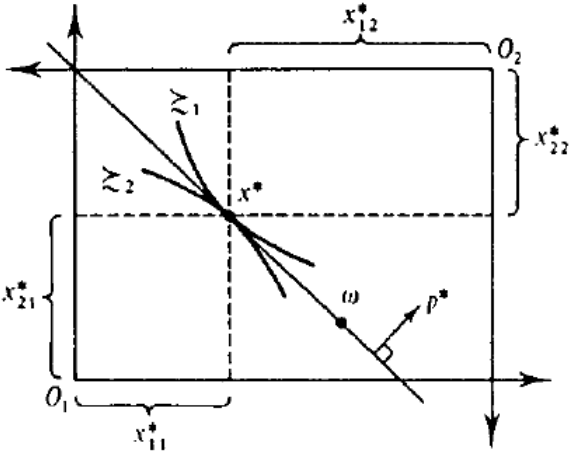
- Note that:
  - By definition x_i maximizes utility subject to the budget constraint: $x_i = x_i(p, p \cdot \omega_i)$ where $p\omega_i = \sum_\ell p_\ell \omega_{\ell i}$ is the value of consumer endowment (wealth) and x_i is the Walrasian demand.
  - By definition of the Edgeworth box, the allocation is non-wasteful and therefore feasible, so the equilibrium market clearing condition is therefore:

$$x_1(p, p \cdot \omega_1) + x_2(p, p \cdot \omega_2) = \bar{\omega}$$

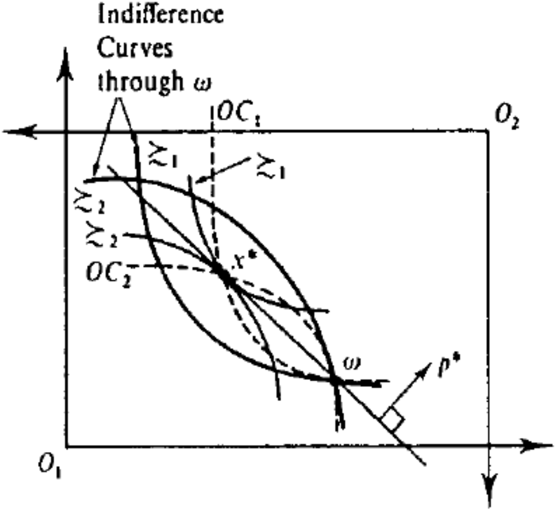
  - In the general case:

$$\sum_i x_i(p, p \cdot \omega_i) = \bar{\omega}$$

    - remember that x_i is a vector of L elements, so we have a market clearing condition for each good.


x^*_{12}
O_2
$\succsim_1$
x^*_{22}
$\succsim_2$
x^*
x^*_{21}
ω
p^*
O_1
x^*_{11}

Equilibrium and offer curves



How to find equilibria for a competitive economy

- Derive or write down (if you know them already) consumers Walrasian demands as functions of: $\omega_{\ell i}'s$ and $p_\ell's$.
- Find out what is the total endowment of each good ℓ.
- Write down the market clearing condition for good ℓ (total endowment = demand) (note, that if $L = 2$, you need only one).
- Solve for the ratio of prices as a function of endowments and know parameters $\rightarrow$ equilibrium prices.
  - Note: as Walrasian demand is homogeneous of degree 1 in prices and income, you will always have the *ratio* of prices and not *levels.* Therefore it is usually easier to set one of the prices to 1 (numeraire) and solve for the other price.
- Substitute the equilibrium price ratio into the Walrasian demand $\rightarrow$ equilibrium consumption levels.

Example

Example

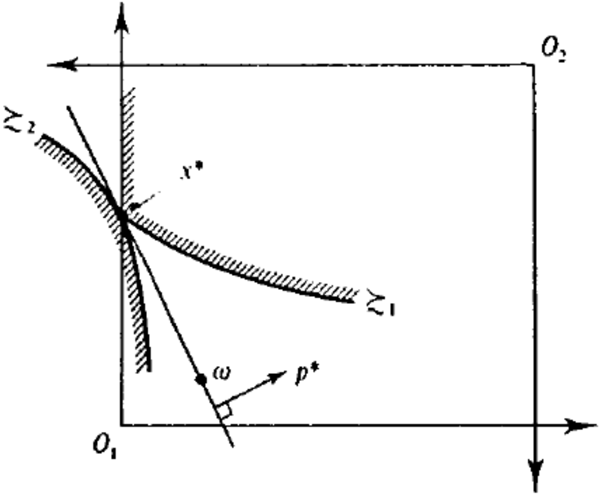
The Cobb-Douglas economy. Suppose each consumer i has the Cobb-Douglas utility function $u_i(x_{1i}, x_{2i}) = x_{1i}^{\alpha} x_{2i}^{1-\alpha}$. Endowments are $\omega_1 = (1, 2)$, $\omega_2 = (2, 1)$.

$$OC_1(p) = \left(\frac{\alpha(p_1 + 2p_2)}{p_1}, \frac{(1-\alpha)(p_1 + 2p_2)}{p_2} \right)$$

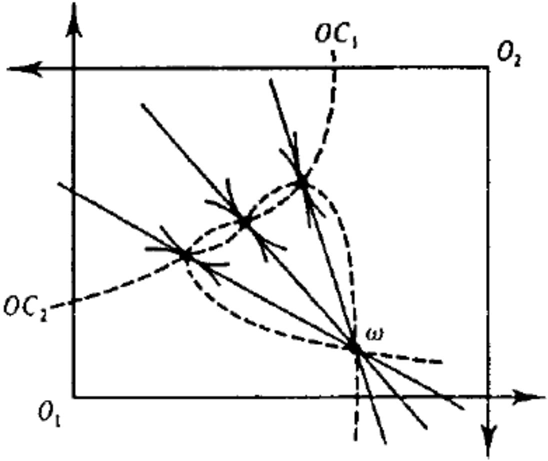
$$OC_2(p) = \left(\frac{\alpha(2p_1 + p_2)}{p_1}, \frac{(1-\alpha)(2p_1 + p_2)}{p_2} \right)$$

- Market clearing: $3 = \frac{\alpha(p_1+2p_2)}{p_1} + \frac{\alpha(2p_1+p_2)}{p_1}$. Find (p_1^*, p_2^*). Solution: $\frac{p_1^*}{p_2^*} = \frac{\alpha}{1-\alpha}$.
- For commodity 2 automatic market clearing.

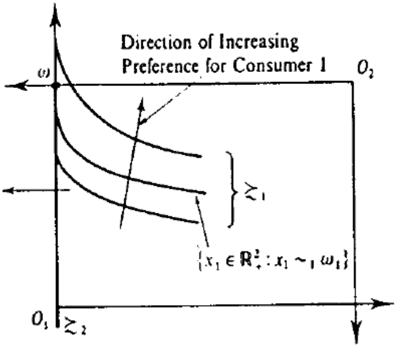
Another example



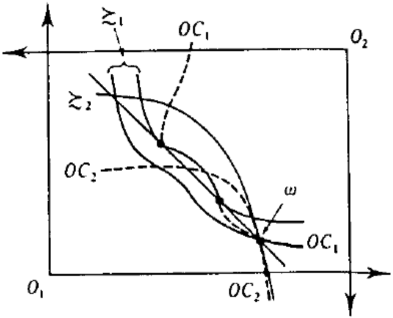
Possibility of multiple equilibria



This happens for $u_1(x_{11}, x_{21}) = x_{11} - \frac{1}{8}x_{21}^{-8}$ and $u_2(x_{12}, x_{22}) = -\frac{1}{8}x_{12}^{-8} + x_{22}$ and $\omega_1 = (2, r)$ and $\omega_2 = (r, 2)$ with $r = 2^{8/9} - 2^{1/9}$. Show that the offer curves intersect in three points.



- Example 1. Consumer 2 has all the good 1. He desires only good 1.
- Consumer 1 has all the good 2. Indifference curve with an infinite slope at ω_1 .
- At $p_2/p_1 > 0$ it is optimal to 2 to consume endowment.
- For 1 at any price it is optimal to purchase a positive amount of good 1.
- 1's demand for good 2 is infinite at $p_2/p_1 = 0$.
- 2's preferences are not strictly increasing



Example 2. Non-convex preferences.

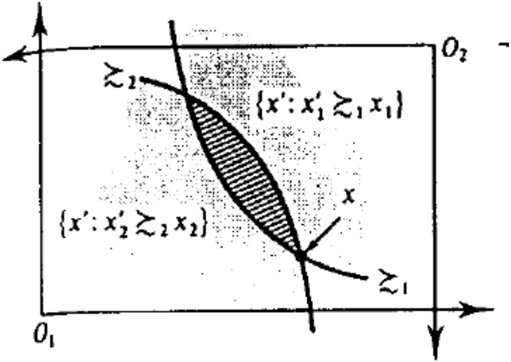
- Consumer 2 has convex preferences.
- Consumer 1 has a non-convex segment on each indifference curve.
- He will never have positive demand on that segment of indifference curve.
- If the offer curve of consumer 2 misses the convex part - non-existence of equilibrium.

- An economic outcome is Pareto optimal (efficient) if there is no alternative feasible outcome at which every individual in the economy is at least as well off and some individual is strictly better off.

Definition

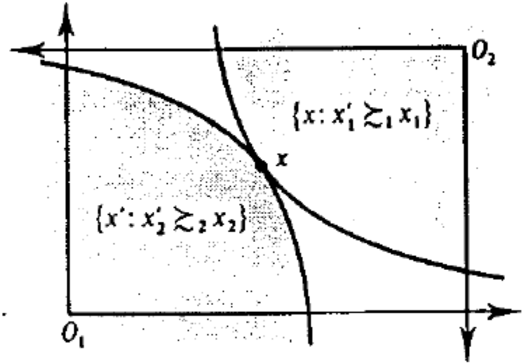
An allocation x in the Edgeworth box is *Pareto optimal* (or Pareto efficient) if there is no other allocation x'in the Edgeworth box with $x_i' \succeq_i x_i$ for all $i = 1, 2$ and $x_i' \succ_i x_i$ for at least one i.

Allocation x not Pareto efficient



Allocation x is Pareto efficient

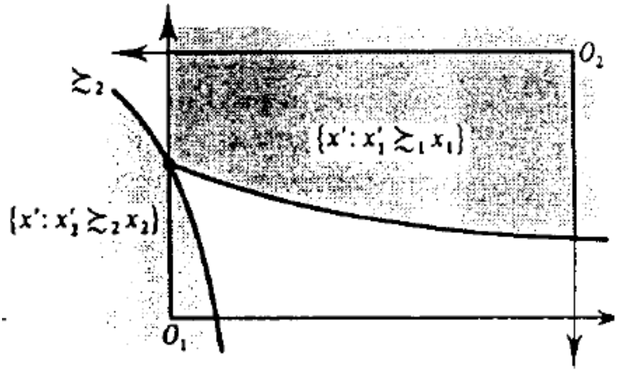
Tangency solution (when equilibrium in the interior of E. box).



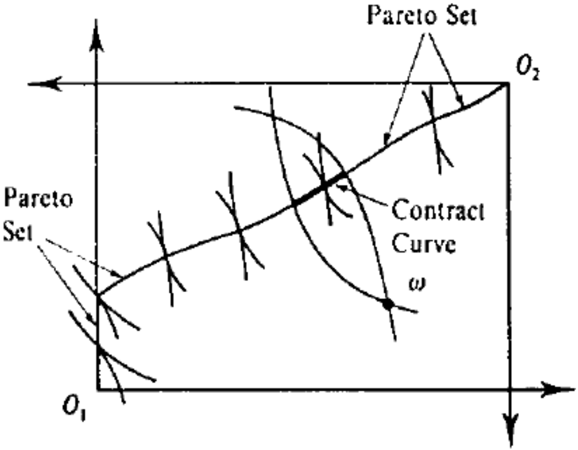
What if the indifference curves are not "smooth"?

Allocation x is Pareto efficient

Tangency does not have to hold.



Pareto set, contract curve



Some facts

Under Walrasian equilibrium:

- Budget line **separates** the two at-least-as-good sets of the two consumers.
- The only point in common is between the two sets is x^* - the Walrasian equilibrium.
- There is no other allocation that can benefit one of the consumers without hurting the other.
  - therefore all Walrasian equilibria belong to the Pareto set.

First fundamental theorem of welfare economics (for a general case)

Theorem

If (x^*, p^*) is a Walrasian equilibrium and preferences are locally non-satiated, then x^* is Pareto optimal.

Proof.

Suppose not. And let x' be another allocation so that $x' \succeq_i x^*$ with strict inequality for at least one i. Then we have:

$$px_i' \geq p\omega_i \text{ with strict inequality at least one } i.$$

Summing over i and using the fact that x' is feasible, we have:

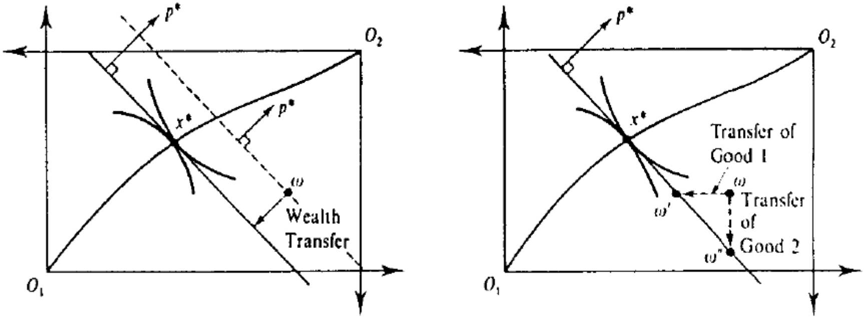
$$p\sum_{i=1}^{I}\omega_i = p\sum_{i=1}^{I}x_i' > \sum_{i=1}^{I}p\omega_i,$$

which is a contradiction.

First fundamental theorem of welfare economics

- Some points:
  - we will see a more general version later.
  - in a perfectly competitive setting any equilibrium is Pareto optimal and the only possible justification of intervention in the economy is fulfilling distributional objectives ("invisible hand").
- Can we go back? Is any Pareto optimal outcome attainable through competitive equilibrium?
- Yes, there is a converse result. However, we need more tools.

Wealth transfers vs. endowment transfers



An equilibrium with transfers

Definition

An allocation x^* in the Edgeworth box is supportable as an *equilibrium with transfers* if there is a price system p^* and wealth transfers T_1 and T_2 satisfying $T_1 + T_2 = 0$, such that for each consumer i we have:

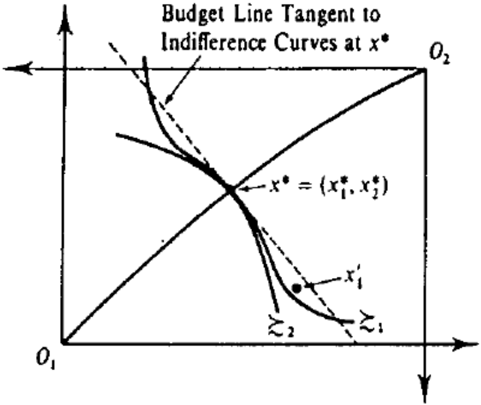
$$x_i^* \succeq_i x_i' \text{ for all } x_i \in \mathbb{R}^2_+ \text{ such that } p^* \cdot x_i' \leq p^* \cdot \omega_i + T_i.$$

Second fundamental theorem of welfare economics

Theorem

If the preferences of the two consumers in an Edgeworth Box are continuous, convex and strictly monotone, then any Pareto optimal allocation is supportable as an equilibrium with transfers.

Failure of 2WT



Second welfare theorem (revealed preference version)

Theorem

Suppose that x^* is a Pareto efficient allocation and that preferences are non-satiated. Suppose further that a competitive equilibrium exists from the initial endowment $\omega_i = x_i^*$ and let it be given by (p', x'). Then in fact, (p', x^*) is a competitive equilibrium.

Proof.

Since x_i^* is in the consumer i budget set by construction, we must have that $x_i' \succeq x_i^*$. Since $x^*$is Pareto efficient by assumption, this implies that $x_i^* \sim_i x_i'$. Thus if x_i' is optimal, so is x_i^*. Therefore (p', x^*) is a competitive equilibrium. □