

Advanced Microeconomics

Consumer theory: preferences, utility, budgets

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Introduction

The plan:

- 1 Some (very basic) definitions
- 2 Preference relations (most general)
- 3 Utility functions
- 4 The consumer optimization problems with utility functions
- 5 Walrasian vs Hicksian demand
- 6 Duality of the consumer problem

Commodities

“The decision problem faced by the consumer in a market economy is to choose consumption levels of various goods and services that are available for purchase in the market”

- **commodities** - goods and services.
- physical characteristics:
 - homogeneous/differentiated
 - divisible/indivisible
 - durable/storable/perishable
- BUT: location, time, state of the world contingency!

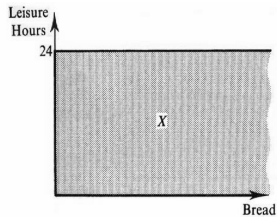
The choice set

- A set of mutually exclusive alternatives that the decision maker can choose from.
 - We will usually denote such set in capital letters: eg. X
 - The choices made by the decision maker we will denote by lower case letters: eg. $x, y \in X$
 - We will refer to X as the **consumption set**
 - We will usually assume $X \subset \mathbb{R}^n$ is nonempty, closed, convex, bounded below ($\exists z \in \mathbb{R}^n$ such that $\forall x \in X, x_i \geq z_i, i = 1, 2, \dots, n$) and has a nonempty interior (but there may be exceptions from that rule)
 - Usually we will restrict attention to $X = \mathbb{R}_+^n$ (example: economic 'bads')

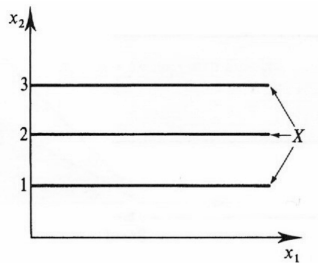
Commodities

- we will refer to goods as finite if we can talk about the total **number** of commodities
- usually refer to a vector $x \in \mathbb{R}^n$
- general assumptions:
 - commodities are divisible
 - there is a price on each commodity set by a market

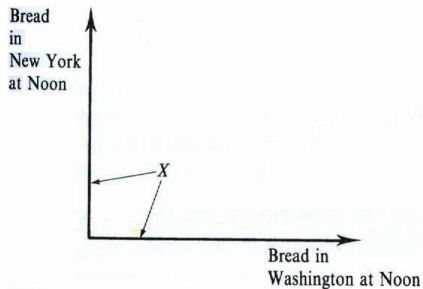
Bounded consumption set



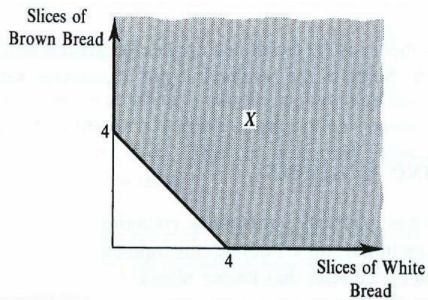
Indivisible commodities



Location



Survival needs



Preference relations

- We will be talking about the consumer/decision maker choices based on his preferences
- In the most general setting, we will talk about *preference relation* \succeq .
- Our decision maker will be comparing different alternatives
 - eg. $A \succeq B$ will mean that A is at least as good as B (“just” preferred)
 - $A \succ B$ will mean that A is **strictly** preferred to B
 - $A \sim B$ will mean that the decision maker is indifferent between A and B .
- The easiest example: $R, \succeq, \succ, =$

Rational preference relation

- *rational preference* relation satisfies:
 - completeness: for all $x, y \in X$ we have that $x \succeq y$ or $y \succeq x$ or both (we can always compare two choices)
 - transitivity: for all $x, y, z \in X$, if $x \succeq y$ and $y \succeq z$, then $x \succeq z$.
- Both are, in fact, fairly restrictive.

Rational preference relation

If \succsim is rational:

- 1 \succ is irreflexive ($\forall x \in X$, not $x \succ x$) and transitive
- 2 \sim is reflexive ($\forall x \in X$, $x \sim x$) and transitive and symmetric
($\forall x, y \in X$, $x \sim y \Rightarrow y \sim x$)
- 3 $x \succ y \succsim z \Rightarrow x \succ z$

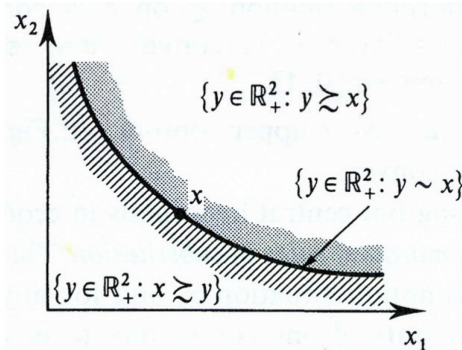
Some more properties

Monotonicity:

- Preference relation is *monotone* if $(x, y \in X$ and $y \gg x)$ implies $y \succ x$.
- Preference relations is *strongly monotone* if $(y \geq x$ and $y \neq x)$ implies $y \succ x$.
- Goods are *desirable* if we prefer more to less.

Properties of preferences

- Indifference set - set of all bundles that are indifferen to a given x :
 $\{x \in X : y \sim x\}$
- Upper contour set - set of all bundles that are at least as good as x :
 $\{x \in X : y \succeq x\}$
- Lower contour set - set of all bundles that x is at least as good as:
 $\{x \in X : x \succeq y\}$

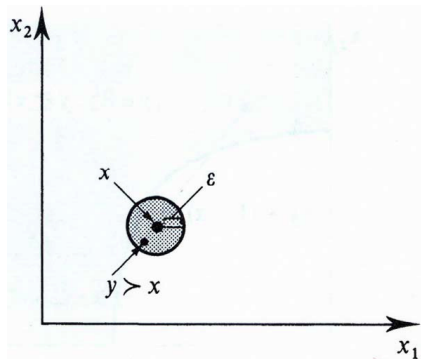


Local nonsatiation

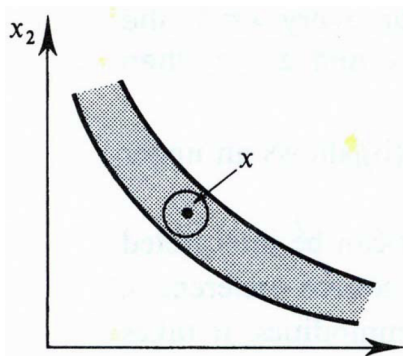
- Preferences are locally nonsatiated on X if for every $x \in X$ and every $\varepsilon > 0$, there is $y \in X$ such that $\|y - x\| \leq \varepsilon$ and $y \succ x$.
- $\|y - x\|$ is the Euclidean distance

$$= \left(\sum_{l=1}^L (y_l - x_l)^2 \right)^{1/2}$$

- In other words: for each x there exists y that is arbitrarily close to x and $y \succ x$.



Local non-satiation violated



Exercise 3.B.1 MWG

Show the following:

- 1 If \succeq is strongly monotone, then it is monotone.
- 2 If \succeq is monotone, then it is locally nonsatiated.

Convexity

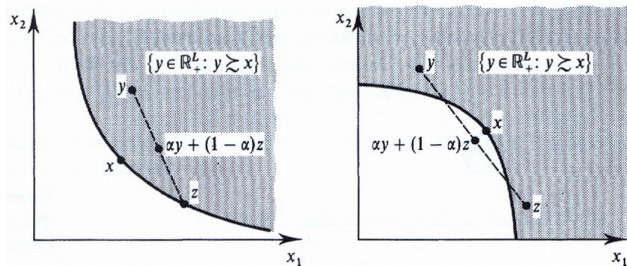
A preference relation is convex if for every $x \in X$ its upper contour set $\{y \in X : y \succeq x\}$ is convex

- define **convexity**: for all $\alpha \in [0, 1]$ $y \succeq x$ and $z \succeq x$ implies that $\alpha y + (1 - \alpha)z \succeq x$
- so every linear combination of y and z preferred to x is also preferred to x

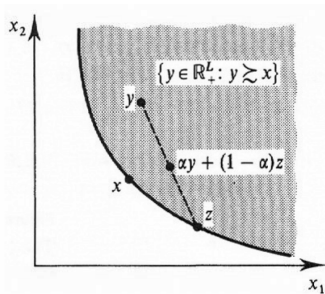
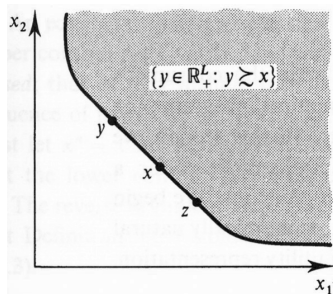
Strict convexity:

- for all $\alpha \in (0, 1)$ $y \succeq x$ and $z \succeq x$ with $y \neq z$ implies that $\alpha y + (1 - \alpha)z \succ x$

Convexity and non-convexity

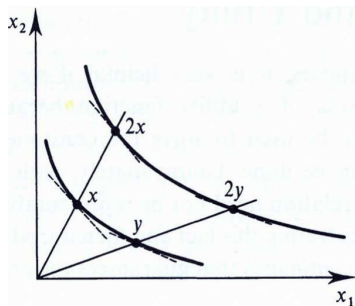


Convexity and strict convexity



Homothecity

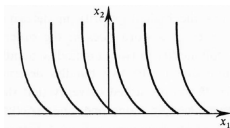
A preference relation is homothetic if all indifference sets are related by proportional expansion along rays from origin; that is if $x \sim y$, then $\alpha x \sim \alpha y$ for any $\alpha > 0$.



Quasi-linearity

A preference relation \succeq on $X = (-\infty, \infty) \times \mathbb{R}_+^{L-1}$ is *quasi-linear* with respect to commodity 1 (the numeraire commodity) if:

- 1 All the indifference sets are parallel displacements of each other along the axis of commodity 1. That is, if $x \sim y$, then $(x + \alpha e_1) \sim (y + \alpha e_1)$ for $e_1 = (1, 0, \dots, 0)$ and any $\alpha > 0$.
- 2 Good 1 is desirable; that is, $x + \alpha e_1 \succ x$ for all x and $\alpha > 0$.



Utility function

To make our life easier (and the problem tractable and implementable in a computable way), we will usually assume that preferences can be described by a utility function.

- A function $u : X \rightarrow \mathbb{R}$ is a *utility function* representing relation \succeq if for all $x, y \in X$:

$$x \succeq y \Leftrightarrow u(x) \geq u(y).$$

- Utility functions are *ordinal*. *Monotonic* transformations do not alter the order. Worn example: Cobb-Douglas utility

Proposition: A preference relation \succeq can be represented by a utility function only if it is rational (proof: MWG).

Can any rational preference relation be represented by a utility function?

The lexicographic preference

Lets limit our attention to \mathbb{R}^2 . Let $x \succeq y$ if either $x_1 > y_1$ or “ $x_1 = y_1$ and $x_2 \geq y_2$ ”. Dictionary sort order.

- Indifference sets are singletons.
- Problem with continuity. Continuity means that that preference relations are preserved under limits.
- Lexicographic preferences are not continuous: sequence of $x^n = (1/n, 0)$, $y^n = (0, 1)$. For every n we have $x^n \succ y^n$. What about limits with $n \rightarrow \infty$?
- Rational preference relation can be represented by a utility function if it is continuous. Continuity assures that we can find a bundle that is indifferent to a given bundle arbitrarily close. Also it assures that for every bundle x we can find an indifference curve. u in this case will also be continuous.
- Therefore we can map from \mathbb{R}^n to \mathbb{R} to provide ordering.

Usual assumptions

- The utility function is continuous (preference relations are continuous).
- The utility function is differentiable (twice continuously).
 - A widely used exception: Leontief preferences:
 - $x'' \succeq x'$ iff $\text{Min}\{x''_1, x''_2\} \geq \text{Min}\{x'_1, x'_2\}$
 - $u(x) = \text{Min}\{x_1, x_2\}$
- The utility function is increasing.

Convexity

- We will usually assume that the functions are **either**
 - quasiconcave (upper contour sets are convex) or
 - strictly quasiconcave (upper contour sets are strictly convex).
- A useful math result (see MWG appendix):
 - a function is quasiconcave if its Hessian matrix is negative semidefinite
 - a function is strictly quasiconcave if its Hessian matrix is negative definite
- See: http://en.wikipedia.org/wiki/Quasiconvex_function
- and even more basic:
http://en.wikipedia.org/wiki/Positive-definite_matrix

Other assumptions

- A homothetic preference relation can be represented by a homogeneous of degree one utility function, such that:
 $u(\alpha x) = \alpha u(x)$
- Quasilinear utility - quasilinear preferences:

$$u(x) = x_1 + \phi(x_2, \dots, x_L)$$

The budget

We described the problem, what the consumer can consume and what his preferences are.

- Now lets turn to the remaining part of the consumer problem: what he can afford.

- The consumer is facing a set of prices: $p = \begin{bmatrix} p_1 \\ \vdots \\ p_L \end{bmatrix} \in \mathbb{R}^L$

- The consumer has a wealth w
- Consumer can afford bundles such that:

$$p \cdot x = p_1 x_1 + \dots + p_L x_L \leq w$$

- We will usually allow only prices >0 (otherwise free goods).
- We will also assume price taking.

Walrasian/competitive budget

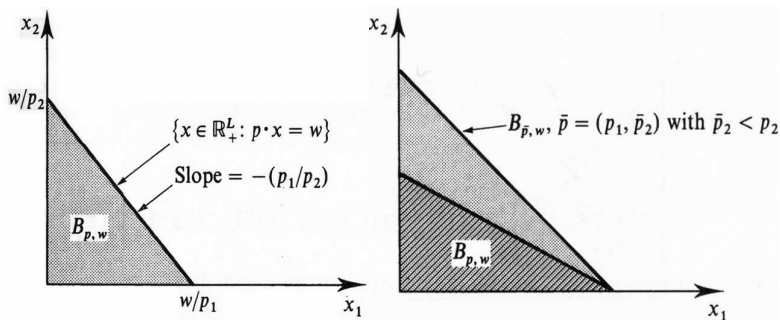
The Walrasian or competitive budget set $B_{p,w} = \{x \in \mathbb{R}_+^L : p \cdot x \leq w\}$ is a set of all feasible consumption bundles for the consumer who faces market prices p and has wealth w .

When all goods are desirable we will mostly talk about cases where the consumer budget constraint is binding (the consumer will be on the upper boundary of the budget set).

- when $L = 2$, we talk about the budget line $p \cdot x = w$
- when $L > 2$ we talk about the budget hyperplane $p \cdot x = w$

Budget set properties

- The budget set is convex (usually)
- The slope shows the market rate of exchange of goods
- Effects of a price change



A real budget set...

