# Homework 2

Let me know if you spot errors in the homework. Good luck.

1. Consider the production function $q = f(z_1, z_2) = z_1^2 + z_2$. What can you say about returns to scale in this case?

2. Consider the production function $q = f(z_1, z_2) = z_1 + z_2$.

   (a) draw some isoquants of this production function.

   (b) derive conditional factor demands (watch out for corner solutions).

   (c) derive the cost function $C(q, w_1, w_2)$.

   (d) draw the cost function for $w_1 = w_2 = 1$ (on the horizontal axis put $q$ and $C(q, 1, 1)$ on the vertical axis).

3. Consider a production function $q = f(z_1, z_2) = z_1^{0.3} z_2^{0.2}$.

   (a) find the optimum level of inputs that maximize profits at wages $(w_1, w_2) = (15, 10)$ and price of the final output $p = 150$.

   (b) find the level of output with the input choices found in part (a).

   (c) what are the profits at that output level?.

4. Given the production function (one output)

$$q = f(z_1, \ldots, z_L) = \prod_{l=1}^{L} z_l^{\alpha_l}$$

   with $\sum_{l=1}^{L} \alpha_l = 1$, where $L$ is the number of inputs.

   (a) What can you say about returns to scale?

   (b) Derive the conditional factor demands $z_l(q, w)$

   (c) Derive the cost function $C(q, w)$ and draw it for $w_1 = w_2 = 1$.

   (d) Derive average and marginal cost functions.

   (e) With such a function, can you determine the profit maximizing level of output? Why?

5. For the Leontief production function (one output) $q = f(z) = \min\{z_1/a_1, \ldots, z_l/a_l, \ldots, z_L/a_L\}$ where $a_l > 0, \forall_{l \in L}$, and $L$ is the number of inputs:

   (a) draw a few isoquants for $L = 2$. Graphically determine the optimal combination of inputs $z_1$ and $z_2$.

   (b) for $L = 2$ derive the conditional factor demands and the cost function.

   (c) what can you say about returns to scale?

   (d) generalize your result in (b) for general $L > 2$. Hint: refer to the similar question in the first homework.

6. For the CES production function:

$$q = f(z_1, z_2) = (z_1^\rho + z_2^\rho)^{1/\rho}.$$

   (a) Find conditional factor demands and the cost function.

   (b) Find the elasticity of substitution (use the approximation in logs).

7. Suppose the two-input production function has corresponding cost function:

$$C(q, w_1, w_2) = (5w_1 + 2w_2)q^2$$

   (a) find conditional factor demands (use Shephard's lemma)

   (b) draw the cost function for $w_1 = w_2 = 1$.

   (c) is the production function homothetic[1]?

   (d) find the profit function $\pi(p, w)$ (you need to do the profit maximization problem using the cost function).

[1] for a function to be homothethic, ratios of optimal factor (input) choices have to be constant with constant wages and varying output

8. Suppose the two-input production function has corresponding cost function:

$$C(q, w_1, w_2) = 2q^2\sqrt{w_1 w_2}$$

   (a) find conditional factor demands.
   (b) draw the total cost function for $w_1 = w_2 = 1$.
   (c) draw the marginal and average cost curves for $w_1 = w_2 = 1$.
   (d) is the production function homothetic?
   (e) find the profit function $\pi(p, w)$.
   (f) find the profit maximizing level of output $q$ when $w_1 = w_2 = 1$ and the price of the final good is $p = 8$. Depict your findings on the graph you have drawn in c). What is the level of profits at that level of output?

9. Suppose the two-input production function has corresponding cost function:

$$C(q, w_1, w_2) = 2qw_1^{0.25}w_2^{0.75}$$

   (a) find conditional factor demands
   (b) is the production function homothetic?
   (c) find the profit function $\pi(p, w)$.
   (d) find the profit maximizing level of output $q$ when $w_1 = w_2 = 1$ for any level of $p > 0$ (think first about returns to scale).

10. Suppose the two-input production function has corresponding cost function:

$$C(q, w_1, w_2) = q^{0.5}w_1^{0.4}w_2^{0.6}$$

   (a) find conditional factor demands
   (b) what are the returns to scale (increasing/decreasing)?
   (c) what is the profit maximizing output at prices $p$ and $w$?
   (d) draw the marginal and average cost functions for $w_1 = w_2 = 1$, compare it to price any price $p$ and comment on your finding in (c).

11. A firm has a profit function of the form:

$$\pi(p, w_1, w_2) = p^2/(8\sqrt{w_1 w_2})$$

   (a) Derive the firm's supply function.
   (b) Derive the firm's factor demands.
   (c) Derive the firm's cost function $C(q, w)$ (Hint: use the fact that: $Profit = pq - cost$ and the firm supply function to get a function of $q$ and $w$ and not $p$).

12. A firm has a production set $Y \in \mathbb{R}^4_+$ such that $(z_1, z_2, q_1, q_2) \in \mathbb{R}^4_+$ is feasible if and only if $q_1^2 + q_2^2 \leq z_1 z_2$ ($q$'s are the 2 outputs and $z$'s are the 2 inputs, so on the transformation frontier $q_1^2 + q_2^2 = z_1 z_2$) :

   (a) Draw the transformation curve between $q_1$ and $q_2$ when $z_1 = 2$, $z_2 = 18$ - note that you have to treat $z_1$ and $z_2$ as fixed so that the relationship will be purely between $q_1$ and $q_2$ and the graph obviously should be in $\mathbb{R}^2$.
   (b) Is the technology consistent with non-increasing returns to scale? Explain.
   (c) What would be profit maximizing outputs $q_1$, $q_2$ if $p_1 = p_2 = 4$, $w_1 = w_2 = 1$, $z_1 = 2$, $z_2 = 18$. Illustrate your answer on the diagram you have drawn in (a).