Intertemporal approach to the balance of payments

In this model we consider an optimal choice of consumer that is related to the present and future consumption. Assuming that our present and future income is given by $Y_1$ and $Y_2$ respectively, a consumer faces a budget constraint in the form of:

$$C_1 + \frac{C_2}{1+r} = \frac{Y_1}{1+r} + \frac{Y_2}{1+r}$$

where $1 + r$ is a market discount factor

At the same time its utility function is:

$$U(c_1, c_2) = u(c_1) + \beta u(c_2)$$

where $1 > \beta > 0$

Parameter $\beta$ is a subjective discount factor. Its value lower than 1 means that in our model we assume that an average consumer always prefers present consumption before future consumption (if he preferred identically present and future consumption $\beta$ would equal 1).

In order to determine the optimal level of consumption in both periods we need to maximize utility function at given budget constraint. Hence, we need to calculate marginal rate of substitution between period 1 and 2:

$$MRS = \frac{MU_1}{MU_2}$$

Moreover, the intertemporal choice is optimal if we cannot get any profit from reallocation of consumption between different periods. This implies that the standard optimum condition for representative consumer takes form of:

$$MRS = 1 + r$$

Therefore, in equilibrium there must be:

$$u'(c_1) = (1 + r) \beta u'(c_2)$$

Let’s now consider a small economy where the sum of income of all consumers in a given period equals production. The production depends on intertemporal production possibility frontier and the slope of budget constraint. In case of closed economy there is no possibility of
trade. So consumption in each period must equal production in each period. This is shown on the diagram below.

Yet, when we open our economy then we get the possibility of trade and we may achieve higher level of utility. Still, we assume that we are small open economy so our country takes the world’s interest rate as given. Since there is more competition on international capital markets than autarky’s capital markets it seems reasonable to believe that world’s interest rate should be lower than autarky’s interest rate. This leads to a change in the slope of budget constraint and a change in production level. Due to the change in the slope of budget constraint we may move to higher indifference curve (hence, get higher utility) where new consumption level in period 1 and period 2 is $C_1^*$ and $C_2^*$ respectively. This case is illustrated below:

Another effect of opening the economy is an appearance of current account deficit in the first period ($C_1 > Y_1$) and a current account surplus in the second one ($C_2 < Y_2$).
Question 1. According to you, which of the following countries has the intertemporal production possibility frontier skewed towards present consumption of goods and which has the production possibility frontier skewed towards future consumption of goods?

a) A country which receives a huge inflow of immigrants (e.g. Argentina or Canada in the XIXth century);

b) A country which is one of the world’s leaders in terms of technology, yet its advantage is decreasing systematically (e.g. Great Britain in the XIXth century or the United Stated today);

c) A country that possess huge oil deposits that can be easily extracted (e.g. Saudi Arabia);

d) A country that possess huge oil deposit, yet they can be extracted only after great investment (e.g. Norway that has oil deposits under the North Sea);

e) A country such as South Korea that is quickly approaching the most developed countries.

Question 2. In a country A that is a small closed economy, intertemporal production possibility frontier is symmetric against the 45 degrees line (maximum level of present and future production is equal). We also know that this economy is in its long term equilibrium and that present production equals exactly a half of the maximum level of present production.

a) Show this situation on the diagram plotting present and future production and consumption, iso-value line that assures production equilibrium and the indifference curve that assures consumption equilibrium;

b) What will happen to the present and future production and consumption if we open the economy and the world’s interest rate is higher than the autarky’s interest rate. Show changes on the diagram. What can we say about the current account balance in present and future?

Question 3. Country A has its intertemporal production possibility frontier skewed towards present consumption while country B has its intertemporal production possibility frontier skewed towards future consumption. Both economies are open economies and their indifference curves are symmetric against the 45 degrees line.

a) Show the situation on the diagrams (each country separately). Plot present and future production and consumption, iso-value line that assures production equilibrium and the indifference curve that assures consumption equilibrium. What can we say about the current account balance in both countries in present and future?
b) What will happen if the future discount factor falls? Show changes on the diagrams.

c) What will happen if the world’s interest rate falls (r*)? Show changes on the diagrams.

In what follows consider a small country inhabited by identical citizens with a two-period lifetime and representative household’s preferences over consumption given by:

\[ U(c_1, c_2) = \log(c_1) + \beta \log(c_2) \quad \text{with} \quad 1 > \beta > 0 \]

Each consumer may allocate its income between consumption and foreign bonds. Hence:

\[ C_1 + B_1 - B_0 = r_0 B_0 + Y_1 \quad \text{where} \quad B \text{ stands for foreign bonds} \]

Finally, we assume that domestic interest rate equals world’s interest rate and that:

\[ CA_i = B_i - B_0 \]

**Question 4.** Assume that the economy starts with no foreign assets (B_0 = 0).

a) Explain why in the equilibrium it has to be that B_2 = 0: Derive the intertemporal budget constraint and show that CA_1 = -CA_2

b) Solve the households' maximization problem. Interpret the first order condition and determine the consumption function in period 1 and period 2.

c) Suppose that Y_1 and Y_2 are such that the economy runs a current account deficit in the first period. Draw a graph of the equilibrium in this case. Show graphically the gains from trade (assuming that the economy was acting under autarky in the first period).

d) Define the autarky interest rate. Explain the underlying reason for which the country is running a current account deficit. (i.e. what is the relationship between autarky and world interest rate?). You can show it graphically or more algebraically.

e) Suppose that \( \beta = \frac{1}{1 + r} \) and that Y_1 = Y_2 = Y. There is a now a government that runs a balanced budget in both periods, i.e. G_i = T_i where i = 1, 2. Determine the new budget constraint of the consumer. Derive what happens to consumption levels in both periods and CA_1 if G_1 > 0 and G_2 = 0. What would happen to consumption and the current account if G_1 = G_2 = G > 0?
Question 5. Assume $\beta = 0.9$. If the country accesses the international capital market it can borrow/lend at interest rate $r$ with $(1+r)^{-1} = 0.9$. Individual endowments are $y_1 = 40$ and $y_2 = y_1(1+g)$ with $g = 0.025$. The country’s net foreign asset position at the beginning of period 1 is $B_0 = 0$.

(a) Assume that because of capital controls the country cannot borrow/lend on the international capital market. Determine the autarky interest rate for the country in question.

(b) Assuming the country is opened to the international capital market, solve for the optimal consumption level in the first period. Will the country borrow or lend in the first period?

(c) Work out the value of the current account balances in period 1 and 2 (Suggestion: work with symbols and plug in the numbers only at the very end).

(d) Suppose the country’s income growth rate $g$ were 0.05 rather than 0.025. Would the first period current account deficit/surplus be larger or smaller?

Question 6. Suppose now that individual endowments are $y_1$ and $y_2$. Assume that endowments are such that the current account is balanced. If the capital stock cannot adjust immediately, a wave of immigration reduces income per capita temporarily by reducing the capital stock per head. You can model this as a fall in $y_1$ with $y_2$ unchanged. In Israel, during the early nineties they saw a big wave of immigration mainly from former Communist countries. What do you expect happened to Israel’s current account?

Question 7. Assume a small country has an initial stock of foreign assets $B_0 < 0$. The latter implies that the stock of assets at the end of the first period is given by $B_1 - B_0 = rB_0 + Y_1 - C_1$. Assume it is $\beta (1+r) = 1$. The first period endowment is $y_1$ and there is no endowment in period 2.

(a) Write down the country’s intertemporal budget constraint and the optimality condition for consumption.

(b) What would be the effect of a debt reduction (an increase in $B_0$) on the present value of the country’s income. What is its effect on consumption and the current account in the first period?