Testing the monetary model of exchange rate determination: new evidence from a century of data

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Abstract

We test the long-run monetary model of exchange rate determination for a collection of 14 industrialized countries using data spanning the late nineteenth or early twentieth century to the late twentieth century. Interestingly, we find support for a simple form of the long-run monetary model in over half of the countries we consider. For these countries, we estimate vector error-correction models to investigate the adjustment process to the long-run monetary equilibrium. In the spirit of Meese and Rogoff [Journal of International Economics 14 (1983) 3–24], we also compare nominal exchange rate forecasts based on monetary fundamentals to those based on a naïve random walk model.

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1. Introduction

The monetary model of exchange rate determination posits a strong link between the nominal exchange rate and a simple set of monetary fundamentals.
The monetary model’s clear-cut intuition—that a country’s price level is determined by its supply and demand for money and that the price level in different countries should be the same when expressed in the same currency—makes it an attractive theoretical tool for understanding fluctuations in exchange rates over time. It also provides a long-run benchmark for the nominal exchange between two currencies and thus a clear criterion for determining whether a currency is significantly “overvalued” or “undervalued.”

Despite its theoretical appeal, the monetary model did not escape the Meese and Rogoff (1983) trap that seemingly ensnared all models of exchange rate determination. In their seminal paper, Meese and Rogoff (1983) find that a naïve random walk model outperforms an array of structural models, including those based on monetary fundamentals, in predicting U.S. dollar exchange rates at horizons of up to 12 months during the late 1970s and early 1980s. Mark (1995) rekindled hope for the monetary model by showing that deviations from a simple set of monetary fundamentals—relative money supplies and relative real output levels—are useful in predicting U.S. dollar exchange rates at longer horizons over the 1981–1991 period. However, Berben and van Dijk (1998) and Berkowitz and Giorgianni (2001) show that Mark’s (1995) findings hinge critically on the assumption of a stable cointegrating relationship among nominal exchange rates, relative money supplies, and relative output levels. When this assumption is relaxed, the evidence for exchange-rate predictability in Mark (1995) is greatly diminished, and, in fact, Mark (1995) fails to find evidence of cointegration among nominal exchange rates and monetary fundamentals in preliminary testing. A number of other studies also find little evidence of cointegration among nominal exchange rates and monetary fundamentals during the post-Bretton Woods float; see, for example, Meese (1986), Baillie and Selover (1987), McNown and Wallace (1989), Baillie and Pecchenino (1991), and Sarantis (1994). The lack of empirical evidence for a stable long-run relationship among nominal exchange rates and monetary fundamentals renders the monetary model a seemingly plausible theoretical model with little practical relevance.

A ready explanation for the failure to find cointegration between nominal exchange rates and monetary fundamentals in much of the extant literature is the

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1Chinn and Meese (1995) also find that monetary fundamentals are helpful in predicting U.S. dollar exchange rates over the 1983–1990 period.
2Chinn and Meese (1995) also fail to find strong evidence of cointegration among nominal exchange rates and monetary fundamentals.
3MacDonald and Taylor (1994) find evidence of cointegration between the U.S. dollar–U.K. pound exchange rate and a set of monetary fundamentals from 1976 to 1990, but their cointegrating vector is difficult to interpret theoretically. They only claim that it “does not, in fact, do great violence to the monetary model.” Cushman (2000) finds evidence of cointegration between the U.S. dollar–Canadian dollar exchange rate and a set of monetary fundamentals during the modern float, but the estimated cointegrating coefficients differ widely from those predicted by the monetary model. Cushman (2000) thus concludes that the U.S.–Canadian data do not support the monetary model.
relatively short spans of data typically employed, which cover only the post-Bretton Woods float. Standard tests take no cointegration as the null hypothesis, and the power to reject this null is extremely low using data from the post-Bretton Woods period alone, which spans 25 years or less. It does not help that the data are often sampled at monthly or quarterly frequencies, as the power of unit root and cointegration tests depends on the data’s span, rather than its frequency (Shiller and Perron, 1985; Hakkio and Rush, 1991). A similar situation exists in the empirical purchasing power parity (PPP) literature. Long-run PPP posits a stable long-run relationship between nominal exchange rates and relative price levels, but empirical support for such a relationship is scant using data from the modern float. Again, this can be attributed to the low power of standard tests for samples as short as the modern float. Given that PPP is a building block of the monetary model, it is not surprising that it is difficult to find evidence of cointegration between nominal exchange rates and monetary fundamentals during the modern float.

Two responses to the problem of low power appear in the PPP literature. First, a number of studies employ panels from the post-Bretton Woods float. As initially shown by Levin and Lin (1992), panel techniques can greatly improve the power of unit root and cointegration tests. Indeed, many panel studies find support for long-run PPP for the post-Bretton Woods era, including Frankel and Rose (1996), Oh (1996), Wu (1996), Papell (1997), and Taylor and Sarno (1998). The second response to low power in the PPP literature is the use of long spans of data, often covering more than a century. For example, Abuaf and Jorion (1990), Glen (1992), Lothian and Taylor (1996, 2000), and Taylor (2001a) all use long spans of data to test long-run PPP. These studies also find considerable support for long-run PPP. Both the panel and long spanning data studies show that deviations from long-run PPP are quite persistent and display near-unit-root behavior, precisely the type of stationary behavior that will be difficult for standard single-country tests to detect for samples as short as the modern float.

In regard to the monetary model, two recent studies by Groen (2000) and Mark and Sul (2001) follow the first response in the PPP literature and test for a stable long-run relationship between nominal exchange rates and monetary fundamentals using panel cointegration tests for the post-Bretton Woods float. Interestingly, these studies both find strong evidence of cointegration among nominal exchange rates, relative money supplies, and relative real output levels using panel cointegration tests. Mark and Sul (2001) actually find support for a very simple long-run monetary model that imposes basic homogeneity restrictions. They also find that nominal exchange rate forecasts based on the monetary model are generally superior to forecasts of a naïve random walk model. Given that the main criticisms of Mark (1995) are based on the lack of cointegration among nominal

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4See Rogoff (1996) and Sarno and Taylor (2001) for recent surveys of the PPP literature.
exchange rates and monetary fundamentals, the recent findings of Groen (2000) and Mark and Sul (2001) again rekindle hope in the ability of monetary fundamentals to track nominal exchange rates.

While Groen (2000) and Mark and Sul (2001) follow the first response in the PPP literature and use panel data from the modern float, no study pursues the second response in the PPP literature and tests the monetary model using long spans of data. In this paper, we pursue this second response. Just as Groen (2000) and Mark and Sul (2001) test the monetary model in a panel framework, motivated by the findings of PPP in panel studies, we test the monetary model using long spans of data, motivated by the findings of PPP in studies utilizing long spans of data. In particular, we apply a battery of unit root and cointegration tests to annual data dating back to the late nineteenth or early twentieth century for 14 industrialized countries in order to test the long-run monetary model of exchange rate determination. By using long spans of data, we are able to side-step some of the problems that potentially plague panel-testing procedures. Of particular concern is the possibility of concluding that all countries in a panel satisfy the long-run monetary model when, in fact, some individual countries are not well characterized by the monetary model.

Our estimation results exhibit considerable support for a simple long-run monetary model of U.S. dollar exchange rate determination for France, Italy, the Netherlands, and Spain; moderate support for Belgium, Finland, and Portugal; and weaker support for Switzerland. For these eight countries, we thus find at least some evidence of a theoretically consistent long-run link between nominal exchange rates and a simple set of monetary fundamentals. Along with Groen (2000) and Mark and Sul (2001), our findings are noteworthy given the lack of empirical support in much of the extant literature for the long-run relationship among exchange rates and monetary fundamentals implied by the monetary model. In contrast, we fail to find support for the long-run monetary model for Australia, Canada, Denmark, Norway, Sweden, and the United Kingdom using long spans of data.

For the countries for which we find support for the simple long-run monetary model, we consider two additional topics. First, we estimate vector-error correction models for nominal exchange rates and monetary fundamentals in order to test for weak exogeneity. This gives us insight into the adjustment process through which the long-run equilibrium relationship between exchange rates and monetary fundamentals is maintained. Second, in the spirit of Meese and Rogoff (1983) and

\[\text{Of course, there is the potential problem of structural instability when using long spans of data. We follow the PPP literature that utilizes long spans of data and assume that the dynamics are relatively stable over the sample period. In order to employ more powerful tests of PPP or the long-run monetary model, we have to assume either a substantial degree of homogeneity across countries (in order to employ panel tests) or relatively stable dynamic processes over long periods (in order to employ long spans of data).}\]
Mark (1995), we compare out-of-sample exchange rate forecasts from a naïve random walk model with forecasts based on monetary fundamentals. In line with the recent work of Berben and van Dijk (1998) and Berkowitz and Giorgianni (2001), we find that there is a close connection between the out-of-sample forecast performance of the monetary model and the weak exogeneity test results.

The rest of the paper is organized as follows. Section 2 presents a simple theoretical monetary model and outlines our testing strategy. Section 3 reports our test results for the long-run monetary model, including unit root and cointegration tests. Section 4 analyzes error-correction models suggested by our cointegration test results. Section 5 compares out-of-sample forecasts of nominal exchange rates based on monetary fundamentals with those of a naïve random walk model. Section 6 summarizes our main findings.

2. Theoretical framework and testing strategy

A number of relationships underlie the basic variant of the monetary model. We emphasize that we have in mind a long-run equilibrium relationship. First, stable money demand functions are assumed for the domestic and foreign countries:

\[ m_t - p_t = \alpha_1 i_t + \alpha_2 y_t, \]  
\[ m_t^* - p_t^* = \alpha_1 i_t^* + \alpha_2 y_t^*, \]  

where \( m \) is the money supply, \( p \) is the price level, \( i \) is the nominal interest rate, and \( y \) is real output (all at time \( t \)). With the exception of the nominal interest rate, lower-case letters denote log-levels. Asterisks denote a foreign variable. Note that the money demand parameters, \( \alpha_1 \) and \( \alpha_2 \), are assumed to be identical in the domestic and foreign countries. In our empirical work below, the U.S. serves as the domestic country. Second, purchasing power parity is assumed:

\[ e_t = p_t^* - p_t, \]  

where \( e \) is the nominal exchange rate measured in the number of units of foreign currency per unit of domestic currency. Solving (1) and (2) for \( p_t \) and \( p_t^* \) and substituting the resulting expressions into (3) yields

\[ e_t = (m_t^* - m_t) - \alpha_1 (i_t^* - i_t) - \alpha_2 (y_t^* - y_t). \]

Finally, the monetary model typically assumes uncovered interest parity:

\[ i_t^* - i_t = E(\Delta e_{t+1} \mid I_t), \]

where \( E(\cdot \mid I_t) \) is the expectations operator conditional on information available at

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\textsuperscript{6}Constant terms are suppressed for expositional convenience.
time $t$. If $e_t$ is $I(0)$ or $I(1)^7$, then $\Delta e_{t+1}$ will be equal to zero in the steady state (abstracting away from any deterministic trend growth in $e_t$), so that $i^*_t = i_t$. This leaves

$$e_t = (m^*_t - m_t) - \alpha_2(y^*_t - y_t). \quad (4)$$

Eq. (4) is a basic form of the monetary model that establishes a long-run relationship between the nominal exchange rate and a simple set of monetary fundamentals. Mark and Sul (2001, p. 32) emphasize that (4) can be viewed as a “generic representation of the long-run equilibrium exchange rate implied by modern theories of exchange rate determination,” as a relationship like (4) can be also derived from the Lucas (1982) and Obstfeld and Rogoff (1995) equilibrium models. Mark (1995) and Mark and Sul (2001) impose the additional restriction that $\alpha_2 = 1$ in (4), yielding the simple form of the monetary model:

$$e_t = (m^*_t - m_t) - (y^*_t - y_t).$$

Testing the long-run monetary model entails testing for the existence of a stable long-run relationship among $e_t$, $m^*_t - m_t$, and $y^*_t - y_t$, or, equivalently, testing whether deviations of $e_t$ from a linear combination of $m^*_t - m_t$ and $y^*_t - y_t$ are stationary. Our first step in testing the basic long-run monetary model is thus to examine the integration properties of $e_t$, $m^*_t - m_t$, and $y^*_t - y_t$ using the unit root tests from Ng and Perron (2000), which have good size and power properties. If $e_t \sim I(0)$, then $m^*_t - m_t$ and $y^*_t - y_t$ must also both be $I(0)$ in order for the nominal exchange rate deviations to be $I(0)^8$. In fact, if $e_t, m^*_t - m_t, y^*_t - y_t \sim I(0)$, this is sufficient to establish the stationarity of nominal exchange rate deviations from any linear combination of the relative money supply and relative output level. If $e_t \sim I(1)$, a necessary condition for the long-run monetary model is that one of, or both of, $m^*_t - m_t$ and $y^*_t - y_t$ also be $I(1)$ (and neither can be integrated of an order greater than one). When $e_t, m^*_t - m_t, y^*_t - y_t \sim I(1)$, the long-run monetary model requires these three variables to be cointegrated, and so we estimate the following cointegrating relationship:

$$e_t = \beta_0 + \beta_1(m^*_t - m_t) + \beta_2(y^*_t - y_t). \quad (5)$$

We estimate (5) using OLS, fully modified OLS (Phillips and Hansen, 1990; FM-OLS), dynamic OLS (Saikkonen, 1991; Stock and Watson, 1993; DOLS), and the multivariate maximum likelihood procedure of Johansen (1988, 1991; JOH-ML). As is now well known, OLS estimates of $\beta_1$ and $\beta_2$ in (5) are super-consistent. However, they are not asymptotically efficient, and the OLS covariance matrix for the estimated coefficients is inappropriate for inference, as it is

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7In our unit root test results reported below, we find that $e_t$ is either $I(0)$ or $I(1)$ for all of the countries we consider.

8An exception is if $e_t \sim I(0)$ but $m^*_t - m_t, y^*_t - y_t \sim CI(1,1)$. 
asymptotically biased. In contrast, the FM-OLS, DOLS, and JOH-ML estimates are asymptotically efficient and yield covariance matrices appropriate for inference. We test for cointegration among \(e_t, m_t^q - m_t,\) and \(y_t^q - y_t,\) using the Phillips and Ouliaris (1990), Hansen (1992), and Shin (1994) single-equation procedures, as well as the Johansen (1988, 1991) system-based procedure, which are based on the OLS, FM-OLS, DOLS, and JOH-ML estimates, respectively. In addition, we test the simple form of the monetary model that implies \(\beta_1 = 1\) and \(\beta_2 = -1\) by testing the stationarity of \(e_t - [(m_t^q - m_t) - (y_t^q - y_t)]\) using the same unit root tests that we use for the individual series, as well as the Horvath and Watson (1995) test for cointegration with a pre-specified cointegrating vector. Note that, for a few countries, our unit root test results indicate that \(e_t, m_t^q - m_t \sim I(1),\) while \(y_t^q - y_t \sim I(0).\) For these countries, we proceed with the cointegration analysis as described above but with \(\beta_2 = 0.\)

3. Monetary model test results

3.1. Data

The data used in this study consist of annual observations for the nominal exchange rate (foreign currency per U.S. dollar), the money supply relative to the U.S., and real GDP relative to the U.S. for 14 countries: Australia, Belgium, Canada, Denmark, Finland, France, Italy, the Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, and the United Kingdom. The nominal exchange rate series are from Taylor (2001a), and the money supply and real GDP series are from Bordo and Jonung (1998) and Bordo et al. (1998). The countries considered are determined by data availability. The data run from the late nineteenth or early twentieth century to the late twentieth century and thus cover a variety of international monetary arrangements, including the classical gold standard, the Bretton Woods era, and the modern float. The exact sample period for each country is reported in the tables below. All variables are measured in log-levels.\(^9\)

3.2. Unit root test results

For the 14 countries considered, we first investigate the integration properties of \(e_t, m_t^q - m_t,\) and \(y_t^q - y_t,\) using the Ng and Perron (2000) DF-GLS and \(MZ\) unit root tests, which are variants of the well-known Dickey and Fuller (1979) and Phillips and Perron (1988) tests, respectively. Both of these tests use GLS-detrending (as in Elliot et al., 1996) in order to maximize power and a modified

\(^9\)For some countries, data are missing for certain series for a few wartime years. We follow Taylor (2001a) and fill in the missing data using linear interpolation. We do not include Germany and Japan, as there are a large number of missing observations for these countries.
information criterion to select the lag truncation parameter in an effort to minimize size distortions. Ng and Perron (2000) find that the DF-GLS and $MZ_e$ statistics have good size and power properties in extensive Monte Carlo simulations.\textsuperscript{10} Table 1 reports the results for the DF-GLS and $MZ_e$ tests for our data. Columns (1) and (5) of Table 1 show the country, time period, variable tested, and whether a linear trend was included in the unit root tests. The inclusion of a linear trend is indicated by visual inspection of the series, as well as formal statistical tests.\textsuperscript{11}

Because different tests yield contradictory results on a few occasions, we designate the variables in Table 1 as “$I(1)$,” “$I(0)$,” or “$I(1)$ or $I(0)$” according to the following simple decision rule. We designate a variable as $I(0)$ if both of the tests reject the null hypothesis of nonstationarity at conventional significance levels or if at least one test rejects at the 5 percent significance level. If neither test rejects the null hypothesis of nonstationarity at conventional significance levels, we designate the variable as $I(1)$. Finally, if only one test rejects at the 10 percent level, we designate the series as $I(1)$ or $I(0)$.

Based on the unit root test results in Table 1, we conclude that all three of the variables, $e_i$, $m_i^\# - m_i$, and $y_i^\# - y_i$, are $I(1)$ for Australia, Belgium, France, Italy, Spain, and the United Kingdom. All three variables are found to be $I(0)$ for the Netherlands. For Finland and Portugal, we find that $e_i$ and $m_i^\# - m_i$ are each $I(1)$, while $y_i^\# - y_i \sim I(0)$. For Denmark and Norway, we find that $e_i \sim I(0)$, while $m_i^\# - m_i \sim I(1)$ ($y_i^\# - y_i$ is inconclusive for Denmark and $y_i^\# - y_i \sim I(0)$ for Norway). For Sweden, our test results indicate that $e_i \sim I(0)$ and $m_i^\# - m_i$, $y_i^\# - y_i \sim I(1)$. Finally, for Canada and Switzerland, our unit root test results are inconclusive for $e_i$, while they indicate that $m_i^\# - m_i$, $y_i^\# - y_i \sim I(1)$.\textsuperscript{12}

Next, we discuss the implications of our unit root test results for testing the long-run monetary model using cointegration procedures. For the Netherlands, all three variables are stationary, so we conclude on the basis of the unit root test results alone that deviations of $e_i$ from any linear combination of $m_i^\# - m_i$ and $y_i^\# - y_i$ are stationary for the Netherlands. For Australia, Belgium, France, Italy, Spain, and the United Kingdom, $e_i$, $m_i^\# - m_i$, and $y_i^\# - y_i$ are all $I(1)$. In the next subsection, we thus proceed to test for a cointegrating relationship among these three variables, as required by the long-run monetary model. Finland and Portugal appear to be an intermediate case, with the nominal exchange rate and the relative money supply being $I(1)$, but with the relative output level being stationary. For these countries, the long-run monetary model requires a cointegrating relationship

\textsuperscript{10}In terms of relative performance, the DF-GLS statistic appears more powerful, while the $MZ_e$ test has better size properties. We use the GAUSS program available from Serena Ng’s home page (http://www2.bc.edu/~ngse/research.html) to generate the DF-GLS and $MZ_e$ statistics.

\textsuperscript{11}All of our specifications include a constant term.

\textsuperscript{12}Additional unit root test results (not reported to conserve space) overwhelmingly indicate that the first differences for every variable and every country are stationary, so that no variable for any country appears $I(2)$.
Table 1
Unit root test results

<table>
<thead>
<tr>
<th>(1) Variable</th>
<th>(2) DF-GLS</th>
<th>(3) MZ</th>
<th>(4) Decision</th>
<th>(5) Variable</th>
<th>(6) DF-GLS</th>
<th>(7) MZ</th>
<th>(8) Decision</th>
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Belgium (1880–1989)

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<th>(12) Decision</th>
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<th>(15) MZ</th>
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Canada (1880–1995)

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Denmark (1885–1995)

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Finland (1911–1995)

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Italy (1880–1995)

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<td>−1.12</td>
<td>−1.12</td>
<td>(1)</td>
<td>$y^* - y$</td>
<td>−1.12</td>
<td>−1.12</td>
<td>(1)</td>
</tr>
</tbody>
</table>

†, *, ** indicate significance at the 10, 5, and 1 percent levels, respectively; (trend) indicates that the test allows for stationarity around a linear trend.

Ng and Perron (2000) one-sided (lower-tail) test of $H_0$: Nonstationarity; 10, 5, and 1 percent critical values equal −1.62, −1.98, and −2.38, respectively; when a linear trend is included, 10, 5, and 1 percent critical values equal −2.62, −2.91, and −3.42, respectively;

Ng and Perron (2000) one-sided (lower-tail) test of $H_0$: Nonstationarity; 10, 5, and 1 percent critical values equal −5.7, −8.1, and −13.8, respectively; when a linear trend is included, 10, 5, and 1 percent critical values equal −14.2, −17.3, and −23.8, respectively.
between only \( e_t \) and \( m_t^* - m_t \), as there are no long-run changes in \( y_t^* - y_t \). In the next subsection, we thus test for cointegration between the nominal exchange rate and the relative money supply for Finland and Portugal. For Denmark, Norway, and Sweden, \( e_t \) is stationary, but \( m_t^* - m_t \sim I(1) \). We can thus conclude on the basis of the unit root test results alone that the long-run monetary model does not hold in these countries over our sample. For Canada and Switzerland, it is difficult to tell whether \( e_t \) is \( I(1) \) or \( I(0) \). If \( e_t \sim I(0) \), we have direct evidence against the long-run monetary model, as it requires \( e_t \sim I(1) \) if \( m_t^* - m_t, y_t^* - y_t \sim I(1) \). For these two countries, we give the monetary model a chance and test for cointegration among \( e_t, m_t^* - m_t \), and \( y_t^* - y_t \) under the assumption that \( e_t \sim I(1) \).

### 3.3. Cointegration test results

We report cointegrating coefficient estimates for 10 countries in Table 2 (excluding the Netherlands, Denmark, Norway, and Sweden on the basis of the unit root test results in Table 1). Column (1) of Table 2 gives the country, sample period, and whether a trend is included in the cointegrating vector. Based on the unit root test results in Table 1, we estimate the cointegrating relationship

\[
e_t = \beta_0 + \beta_1 (m_t^* - m_t) + \beta_2 (y_t^* - y_t)
\]

for Australia, Belgium, Canada, France, Italy, Spain, Switzerland, and the United Kingdom, while we estimate the cointegrating relationship

\[
e_t = \beta_0 + \beta_1 (m_t^* - m_t)
\]

for Finland and Portugal. Table 2 includes OLS, FM-OLS, DOLS, and JOH-ML estimates. Following the applications in Hansen (1992), we use the quadratic kernel and the Andrews (1991) automatic bandwidth selector with Andrews and Monohan (1992) prewhitening when computing the FM-OLS estimates. Following Stock and Watson (1993), we set the number of leads and lags in the DOLS estimator equal to two, and we use an autoregressive procedure to compute robust standard errors. We also report a Stock and Watson (1993) Wald test (SW-Wald) of the joint hypothesis that \( \beta_1 = 1 \) and \( \beta_2 = -1 \), as implied by the simple monetary model that sets the common income elasticity of money demand to unity (see

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**As noted above, an exception is when \( e_t \sim I(0) \) but \( m_t^* - m_t, y_t^* - y_t \sim CI(1,1) \). We do not find any evidence of this.**

**We include a trend in the cointegrating vector if the deviations of the exchange rate from the monetary fundamentals exhibit a strong trend that is confirmed by formal statistical tests of the significance of the linear trend in the cointegrating vector. Note that for the JOH-ML estimates in Table 2 and the trace test in Table 3 for France and Switzerland, we restrict the linear trend to appear only in the cointegrating vector (case 2* in Osterwald-Lenum, 1992). A linear trend in the cointegrating vector allows for a deterministic Balassa–Samuelson effect in real exchange rates. However, the inclusion of a linear trend could be viewed as a weaker form of the long-run monetary model.**

**We use the GAUSS program available from Bruce Hansen’s home page (http://www.ssc.wisc.edu/~bhansen/) to generate the FM-OLS estimates.**
Table 2
Cointegrating coefficient estimates, $e_t = \beta_0 + \beta_1 (m_t^* - m_{t-1}) + \beta_2 (y_t^* - y_{t-1})$

<table>
<thead>
<tr>
<th>Country</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
<th>(9)</th>
<th>(10)</th>
<th>(11)</th>
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</thead>
<tbody>
<tr>
<td></td>
<td>OLS estimates</td>
<td>FM-OLS estimates</td>
<td>DOLS estimates</td>
<td>JOH-ML estimates</td>
<td>JOH-chi²</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\beta_0$</td>
<td>$\beta_1$</td>
<td>$\beta_0$</td>
<td>$\beta_1$</td>
<td>$\beta_0$</td>
<td>$\beta_1$</td>
<td>SW-Wald</td>
<td>$\beta_0$</td>
<td>$\beta_1$</td>
<td></td>
<td></td>
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<td>Australia</td>
<td>0.50</td>
<td>-0.14</td>
<td>0.62</td>
<td>-0.21</td>
<td>0.45</td>
<td>-0.19</td>
<td>2.59</td>
<td>0.45</td>
<td>-0.41</td>
<td>2.32</td>
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<tr>
<td>(1880–1995)</td>
<td>(0.09)</td>
<td>(0.21)</td>
<td>(0.29)</td>
<td>(0.63)</td>
<td>(0.35)</td>
<td>(0.73)</td>
<td>(0.30)</td>
<td>(0.63)</td>
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<tr>
<td>Belgium</td>
<td>0.90</td>
<td>-1.10</td>
<td>1.04</td>
<td>-1.06</td>
<td>1.00</td>
<td>-1.01</td>
<td>0.01</td>
<td>0.93</td>
<td>-1.06</td>
<td>0.06</td>
<td></td>
</tr>
<tr>
<td>(1880–1989)</td>
<td>(0.09)</td>
<td>(0.08)</td>
<td>(0.38)</td>
<td>(0.34)</td>
<td>(0.25)</td>
<td>(0.23)</td>
<td>(0.25)</td>
<td>(0.23)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Canada</td>
<td>0.17</td>
<td>0.04</td>
<td>0.23</td>
<td>0.05</td>
<td>0.13</td>
<td>0.10</td>
<td>179**</td>
<td>0.20</td>
<td>0.08</td>
<td>14.04**</td>
<td></td>
</tr>
<tr>
<td>(1880–1995)</td>
<td>(0.03)</td>
<td>(0.04)</td>
<td>(0.13)</td>
<td>(0.15)</td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.08)</td>
<td>(0.08)</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td>1.05</td>
<td>-</td>
<td>1.07</td>
<td>-</td>
<td>1.01</td>
<td>-</td>
<td>0.01</td>
<td>0.82</td>
<td>-</td>
<td>3.10†</td>
<td></td>
</tr>
<tr>
<td>(1911–1995)</td>
<td>(0.03)</td>
<td>-</td>
<td>(0.13)</td>
<td>-</td>
<td>(0.07)</td>
<td>-</td>
<td>(0.11)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>France (trend)</td>
<td>1.03</td>
<td>-1.34</td>
<td>1.09</td>
<td>-1.06</td>
<td>1.03</td>
<td>-1.16</td>
<td>0.28</td>
<td>1.03</td>
<td>-0.09</td>
<td>2.15</td>
<td></td>
</tr>
<tr>
<td>(1880–1989)</td>
<td>(0.04)</td>
<td>(0.10)</td>
<td>(0.11)</td>
<td>(0.25)</td>
<td>(0.10)</td>
<td>(0.31)</td>
<td>(0.14)</td>
<td>(0.54)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>0.99</td>
<td>-0.93</td>
<td>0.95</td>
<td>-1.20</td>
<td>0.96</td>
<td>-1.34</td>
<td>2.55</td>
<td>0.98</td>
<td>-1.87</td>
<td>11.36**</td>
<td></td>
</tr>
<tr>
<td>(1880–1995)</td>
<td>(0.01)</td>
<td>(0.09)</td>
<td>(0.03)</td>
<td>(0.20)</td>
<td>(0.03)</td>
<td>(0.39)</td>
<td>(0.02)</td>
<td>(0.25)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Portugal</td>
<td>1.13</td>
<td>-</td>
<td>1.09</td>
<td>-</td>
<td>1.07</td>
<td>-</td>
<td>1.31</td>
<td>0.94</td>
<td>-</td>
<td>0.47</td>
<td></td>
</tr>
<tr>
<td>(1890–1995)</td>
<td>(0.04)</td>
<td>-</td>
<td>(0.09)</td>
<td>-</td>
<td>(0.06)</td>
<td>-</td>
<td>(0.09)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Spain</td>
<td>0.88</td>
<td>-1.21</td>
<td>0.83</td>
<td>-1.27</td>
<td>0.86</td>
<td>-1.29</td>
<td>11.94**</td>
<td>0.85</td>
<td>-1.05</td>
<td>10.16**</td>
<td></td>
</tr>
<tr>
<td>(1901–1995)</td>
<td>(0.03)</td>
<td>(0.09)</td>
<td>(0.06)</td>
<td>(0.22)</td>
<td>(0.05)</td>
<td>(0.16)</td>
<td>(0.04)</td>
<td>(0.14)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Switzerland (trend)</td>
<td>0.45</td>
<td>-0.79</td>
<td>1.46</td>
<td>-2.08</td>
<td>0.86</td>
<td>-1.30</td>
<td>1.54</td>
<td>1.59</td>
<td>-2.42</td>
<td>17.77**</td>
<td></td>
</tr>
<tr>
<td>(1880–1995)</td>
<td>(0.15)</td>
<td>(0.15)</td>
<td>(0.55)</td>
<td>(0.53)</td>
<td>(0.46)</td>
<td>(0.43)</td>
<td>(0.52)</td>
<td>(0.61)</td>
<td></td>
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<tr>
<td>United Kingdom</td>
<td>0.41</td>
<td>-0.97</td>
<td>0.39</td>
<td>-0.85</td>
<td>0.45</td>
<td>-0.99</td>
<td>22.24**</td>
<td>0.31</td>
<td>-0.71</td>
<td>10.42**</td>
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<tr>
<td>(1880–1995)</td>
<td>(0.04)</td>
<td>(0.04)</td>
<td>(0.14)</td>
<td>(0.16)</td>
<td>(0.19)</td>
<td>(0.24)</td>
<td>(0.13)</td>
<td>(0.16)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

†, *, ** indicate significance at the 10, 5, and 1 percent levels, respectively; for Finland and Portugal, $\beta_2$ is constrained to be equal to zero; standard errors for the coefficient estimates are given in parentheses; (trend) indicates that a linear trend is included in the cointegrating relationship.

* Stock and Watson (1993) one-sided (upper-tail) test of $H_0: \beta_1 = 1, \beta_2 = -1$; 10, 5, and 1 percent critical values for a $\chi^2(2)$ equal 4.61, 5.99, and 9.21, respectively; for Finland and Portugal, the test is for $H_0: \beta_1 = 1$; for Finland and Portugal, the 10, 5, and 1 percent critical values for a $\chi^2(1)$ equal 2.71, 3.84, and 6.64, respectively.

Johansen (1991) one-sided (upper-tail) test of $H_0: \beta_1 = 1, \beta_2 = -1$; 10, 5, and 1 percent critical values for a $\chi^2(2)$ equal 4.61, 5.99, and 9.21, respectively; for Finland and Portugal, the test is for $H_0: \beta_1 = 1$; for Finland and Portugal, the 10, 5, and 1 percent critical values for a $\chi^2(1)$ equal 2.71, 3.84, and 6.64, respectively.
column (8) of Table 2). We select the lag order for the JOH-ML estimator by sequentially testing a VAR in levels using the Sims (1980) modified likelihood-ratio statistic, a maximum lag order of five, and the 10 percent significance level.\footnote{We obtain the following lag orders for the VAR in levels: Australia, 5; Belgium, 4; Canada, 2; Denmark, 5; Finland, 4; France, 2; Italy, 3; Norway, 3; Portugal, 4; Spain, 4; Sweden, 2; Switzerland, 2; United Kingdom, 2. Box–Ljung $Q$-statistics give no indication of serial correlation in any of the VAR equations for these lag orders.}

We also present a $\chi^2$ test due to Johansen (1991), labeled JOH-$\chi^2$, that we use to test the joint null hypothesis that $\beta_1 = 1$ and $\beta_2 = -1$ in the cointegrating vector (see column (11) of Table 2).

For Belgium, Italy, and Spain, all four estimation procedures generally yield parameter estimates close to the theoretical values implied by the simple monetary model ($\beta_1 = 1$ and $\beta_2 = -1$).\footnote{Recall that the OLS standard errors cannot be used for valid inference.} The cointegrating coefficient estimates for Belgium are very close to those implied by the simple monetary model, and the SW-Wald and JOH-$\chi^2$ tests cannot reject the joint restriction that $\beta_1 = 1$ and $\beta_2 = -1$ for Belgium. For Italy, the SW-Wald test also cannot reject the joint restriction that $\beta_1 = 1$ and $\beta_2 = -1$. For Spain, the joint restriction is rejected by both the SW-Wald and JOH-$\chi^2$ tests, apparently due to $\beta_i$ estimates that are significantly less than one. While less than one in statistical terms, the $\beta_i$ estimates are still relatively close to their theoretical value of unity in magnitude. On the whole, the estimated cointegrating relationships for Belgium, Italy, and Spain are consistent with the simple long-run monetary model.

The OLS, FM-OLS, and DOLS coefficient estimates for France are also very close to those implied by the simple monetary model, and the SW-Wald test cannot reject the null hypothesis that $\beta_1 = 1$ and $\beta_2 = -1$. The JOH-ML estimator for France yields a $\beta_1$ estimate very close to unity, and while the estimate for $\beta_2$ has the correct sign, it is quite small in magnitude and statistically insignificant.\footnote{Nevertheless, the JOH-$\chi^2$ test cannot reject the joint restriction that $\beta_1 = 1$ and $\beta_2 = -1$ for France.} Phillips (1994) provides a possible explanation for the discrepancy between the JOH-ML $\beta_2$ estimate and the FM-OLS and DOLS $\beta_2$ estimates. Phillips (1994) shows that cointegration coefficient estimates based on reduced rank regressions (such as JOH-ML) can have Cauchy-like tails and no finite integer moments in finite samples, so that outliers can be expected to occur more frequently than for other asymptotically efficient estimators such as the FM-OLS and DOLS estimators.\footnote{In Monte Carlo experiments, Stock and Watson (1993) also find that outliers can be expected to occur more often for the JOH-ML estimator than the FM-OLS and DOLS estimators.} The JOH-ML $\beta_2$ estimate appears to be an outlier for France.

Turning to the results for Switzerland in Table 2, we see some support for the monetary model. The FM-OLS estimates are reasonably close, and the DOLS estimates are very close, to the values implied by the simple monetary model. Using the SW-Wald test, we cannot reject the null hypothesis that $\beta_1 = 1$ and $\beta_2 = -1$. However, the JOH-ML $\beta_2$ estimate appears to be an outlier for Switzerland.
Using data from the modern float, Cushman (2000) also finds that estimated cointegrating coefficients for Canada do not accord with the monetary model.

Note that we use a longer sample for Portugal in Table 2 than in Table 1. Real output data is only available beginning in 1929 for Portugal, so we use a sample beginning in 1929 for Portugal in Table 1. We use a sample beginning in 1890 for Portugal in Table 2, as nominal exchange rate and money supply data are available beginning in 1890 for Portugal. The unit root test results reported in Table 1 for the nominal exchange rate and relative money supply for Portugal are qualitatively unchanged if we use a sample beginning in 1890.

In the working paper antecedent to the present paper, we test the stability of the cointegrating vectors and find little evidence of structural change for Belgium, Finland, Italy, Portugal, Spain, and Switzerland. There is more evidence for France.

Given the well-documented potential for size distortions when using asymptotic critical values for the trace test (see, for example, Cheung and Lai, 1993), we also calculated bootstrapped p-values. The unreported results indicate that inferences for the trace test are largely unchanged.
Table 3
Cointegration test results, $e_t = \beta_0 + \beta_1 (m_t \pi - m_t) + \beta_2 (y_t \pi - y_t)$

<table>
<thead>
<tr>
<th>Country (Year)</th>
<th>(2) PO-Zn</th>
<th>(3) Trace</th>
<th>(4) $L_n$</th>
<th>(5) $C_n$</th>
<th>Country (Year)</th>
<th>(7) PO-Zn</th>
<th>(8) Trace</th>
<th>(9) $L_n$</th>
<th>(10) $C_n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium (1880–1989)</td>
<td>-20.41</td>
<td>16.12</td>
<td>0.20</td>
<td>0.19*</td>
<td>Portugal (1890–1995)</td>
<td>-12.76</td>
<td>21.13**</td>
<td>0.18</td>
<td>0.16</td>
</tr>
<tr>
<td>Finland (1911–1995)</td>
<td>-23.89*</td>
<td>16.62*</td>
<td>0.21</td>
<td>0.24†</td>
<td>Spain (1901–1995)</td>
<td>-25.71†</td>
<td>35.94**</td>
<td>0.07</td>
<td>0.08</td>
</tr>
<tr>
<td>France (trend) (1880–1989)</td>
<td>-29.59†</td>
<td>41.36†</td>
<td>0.28</td>
<td>0.07</td>
<td>Switzerland (trend) (1880–1995)</td>
<td>-18.05</td>
<td>36.95</td>
<td>0.24</td>
<td>0.11*</td>
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<tr>
<td>Italy (1880–1995)</td>
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<td>53.60**</td>
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<td>0.09</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*,**, indicate significance at the 10, 5, and 1 percent levels, respectively; for Finland and Portugal, $\beta_2$ is constrained to be equal to zero; (trend) indicates that a linear trend is included in the cointegrating relationship.

Phillips and Ouliaris (1990) one-sided (lower-tail) test of $H_0$: No cointegration; 10, 5, and 1 percent critical values equal 22.7, 26.7, and 35.2, respectively; for Finland and Portugal, the 10, 5, and 1 percent critical values equal 17.1, 20.6, and 28.3, respectively; for France, Sweden, and Switzerland, the 10, 5, and 1 percent critical values equal 28.5, 32.8, and 42.0, respectively.

Johansen (1991) one-sided (upper-tail) test of $H_0$: No cointegration; Osterwald-Lenum (1992) 10, 5, and 1 percent critical values equal 26.79, 29.68, and 35.65, respectively; for Finland and Portugal, the 10, 5, and 1 percent critical values equal 13.33, 15.41, and 20.04, respectively; for France and Switzerland, the 10, 5, and 1 percent critical values equal 39.06, 42.44, and 48.45, respectively.

Hansen (1992) one-sided (upper-tail) test of $H_0$: Cointegration; significance is based on $p$-values reported in Hansen (1992).

Shin (1994) one-sided (upper-tail) test of $H_0$: Cointegration; 10, 5, and 1 percent critical values equal 0.163, 0.221, and 0.380, respectively; for Finland and Portugal, the 10, 5, and 1 percent critical values equal 0.231, 0.314, and 0.533, respectively; for France, Sweden, and Switzerland, the 10, 5, and 1 percent critical values equal 0.081, 0.101, and 0.150, respectively.
maintained hypothesis, the null of cointegration may be more appropriate than the null of no cointegration. The Hansen (1992) $L_c$ statistic is based on the FM-OLS residuals, while the Shin (1994) $C_{L}$ statistic is constructed from the DOLS residuals. For France, Italy, and Spain, all four tests indicate the existence of a cointegrating relationship at conventional significance levels, and cointegration is indicated by three of the four tests for Finland and Portugal. The $L_c$ statistic points to cointegration for Belgium and Switzerland.

As the estimated cointegrating coefficients are close to $\beta_1 = 1$ and $\beta_2 = -1$ in Table 2 for Belgium, Finland, France, Italy, Portugal, Spain, and Switzerland, a complementary test of the simple long-run monetary model for these countries is to test whether $e_t = (m_t^* - m_t) - (y_t^* - y_t)$, the deviation of the exchange rate from the level predicted by the simple monetary model, is stationary. This is tantamount to testing for cointegration between the exchange rate and the monetary fundamentals with pre-specified cointegrating coefficients of $\beta_1 = 1$ and $\beta_2 = -1$. We test the stationarity of the deviations using the two unit root tests used for the individual series in Table 1, and the results are reported in Table 4. Table 4 also reports results for the multivariate Horvath and Watson (1995) test of the null hypothesis of no cointegration against the alternative hypothesis of cointegration with pre-specified cointegrating coefficients $\beta_1 = 1$ and $\beta_2 = -1$. As discussed in Horvath and Watson (1995), this test is potentially more powerful than univariate unit root tests when testing for cointegration with a known cointegrating vector. Note that we consider the deviation $e_t = (m_t^* - m_t)$ for Finland and Portugal, in line with the previous results. Also note that we include the deviation $e_t = (m_t^* - m_t) - (y_t^* - y_t)$ for the Netherlands in Table 4. The unit root tests in Table 1 clearly indicate that each component of the deviation is stationary for the Netherlands, and so we expect the deviation to be stationary. We include the Netherlands as a robustness check of the Table 1 results. For Belgium, Italy, the Netherlands, and Spain, the DF-GLS and $MZ_a$ tests both indicate that the deviation is stationary at conventional significance levels. The Horvath and Watson (1995) test supports the simple long-run monetary model for six of the seven countries for which it is relevant. (It is not relevant for the Netherlands.)

Fig. 1 presents graphs of the nominal exchange rate deviations from the level predicted by the simple monetary model for the eight countries examined in Table 4. Vertical lines are drawn for the years 1913, 1946, and 1970 to roughly depict different international exchange rate regimes: classical gold standard, interwar period, Bretton Woods era, and the modern float. A tendency for mean-reversion

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24We use a Bartlett kernel (as in Shin, 1994) and set the lag truncation to four in calculating $C_{L}$.

25We implement the Horvath and Watson (1995) test using a GAUSS program available from Mark Watson’s home page (http://www.wws.princeton.edu/~mwatson/). Critical values for the case where a linear trend is included in the cointegrating vector are not available in Horvath and Watson (1995), so we base inferences on bootstrapped critical values.

26We are following the divisions used in Taylor (2001a).
Table 4

<table>
<thead>
<tr>
<th>(1) Country</th>
<th>(2) DF-GLS*</th>
<th>(3) $M_Z$*</th>
<th>(4) HW*</th>
<th>(5) Country</th>
<th>(6) DF-GLS</th>
<th>(7) $M_Z$</th>
<th>(8) HW</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium (1880–1989)</td>
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<td>-7.60†</td>
<td>9.98†</td>
<td>Netherlands (1900–1992)</td>
<td>-2.17*</td>
<td>-8.62*</td>
<td>-</td>
</tr>
<tr>
<td>Finland (1911–1995)</td>
<td>-1.44†</td>
<td>-4.09</td>
<td>10.03†</td>
<td>Portugal (1890–1995)</td>
<td>0.91</td>
<td>1.86</td>
<td>16.49**</td>
</tr>
</tbody>
</table>

†,*,** indicate significance at the 10, 5, and 1 percent levels, respectively; for Finland and Portugal, the unit root tests are for $e_t - (m^*_t - m_t) - (y^*_t - y_t)$; (trend) indicates that the test allows for stationarity around a linear trend.

*Ng and Perron (2000) one-sided (lower-tail) test of $H_0$: Nonstationarity; 10, 5, and 1 percent critical values equal -1.62, -1.98, and -2.58, respectively; when a linear trend is included, 10, 5, and 1 percent critical values equal -2.62, -2.91, and -3.42, respectively.

Ng and Perron (2000) one-sided (lower-tail) test of $H_0$: Nonstationarity; 10, 5, and 1 percent critical values equal -5.7, -8.1, and -13.8, respectively; when a linear trend is included, 10, 5, and 1 percent critical values equal -14.2, -17.3, and -23.8, respectively.

Horvath and Watson (1995) one-sided (upper-tail) test of $H_0$: No cointegration among $e_t, m^*_t - m_t, y^*_t - y_t$ vs. $H_1$: Cointegration with prespecified cointegrating relationship $e_t = \beta_0 + (m^*_t - m_t) - (y^*_t - y_t)$; 10, 5, and 1 percent critical values equal 9.72, 11.62, and 15.41, respectively; 10, 5, and 1 percent bootstrapped critical values for France (Switzerland) equal 12.89 (12.70), 15.39 (14.60), and 20.16 (17.04), respectively; for Finland and Portugal, one-sided (upper-tail) test of $H_0$: No cointegration among $e_t, m^*_t - m_t$ vs. $H_1$: Cointegration with prespecified cointegrating relationship $e_t = \beta_0 + (m^*_t - m_t)$; for Finland and Portugal, 10, 5, and 1 percent critical values equal 8.30, 10.18, and 13.73, respectively.

In each deviation is evident in Fig. 1. (For France and Switzerland, the deviations appear stationary around a trend.) It is also evident from Fig. 1 that deviations from the monetary fundamentals can be quite substantial and persistent. Because of this, it will be difficult to detect the long-run relationship between the nominal exchange rate and monetary fundamentals using data from the modern float alone.27 Also note that the early 1980s stand out in Fig. 1 as a period where U.S. dollar exchange rates diverge considerably from the underlying monetary fundamentals. The U.S. dollar appears substantially overvalued during this period, as is widely believed.

To summarize the results of this section, we find considerable support for a very simple form of the long-run monetary model of U.S. dollar exchange rate determination for France, Italy, the Netherlands, and Spain using long data spans.

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27Taylor (2001b) argues that it may also be difficult to detect equilibrium relationships if the adjustment to the equilibrium occurs more quickly than the frequency of the available data.
Fig. 1. Deviations from the simple monetary model. Note: vertical lines appear at 1913, 1946, and 1970.
We find moderate support for Belgium, Finland, and Portugal and weaker support for Switzerland. From Fig. 1, we see that long spans of data will generally be required to detect the long-run equilibrium relationship implied by the monetary model.

4. Error-correction models

In order to gain insight into how the long-run equilibrium is restored between nominal exchange rates and monetary fundamentals, we estimate the following bivariate vector error-correction model (VECM) in \( e_t \) and \( f_t \), where \( f_t = (m_t^p - m_t) - (y_t^p - y_t) \):

\[
\Delta e_t = \gamma_0 + \sum_{i=1}^{p} \gamma_{1i} \Delta e_{t-i} + \sum_{i=1}^{p} \gamma_{2i} \Delta f_{t-i} + \lambda_{e,t} \Delta z_{t-1} + \varepsilon_{1t},
\]

where \( z_t = e_t - f_t \). We estimate the VECM for all of the countries in Table 4, with the exception of the Netherlands, due to the stationarity of the nominal exchange rate and monetary fundamentals in the Netherlands. Following the lead of Tables 2–4, we use \( f_t = m_t^p - m_t \) for Finland and Portugal.

Table 5 reports OLS estimates of the error-correction coefficients, \( \lambda_{e,t} \) and \( \lambda_{f,t} \), that govern the adjustment to the long-run equilibrium. For Belgium, Finland, and Italy, the error-correction coefficient in the exchange rate equation \( (\lambda_{e,t}) \) is significant, while the error-correction coefficient in the fundamentals

<table>
<thead>
<tr>
<th>Country</th>
<th>(1) ( \lambda_{e,t} )</th>
<th>(2) ( \lambda_{e,t} )</th>
<th>Country</th>
<th>(4) ( \lambda_{e,t} )</th>
<th>(5) ( \lambda_{e,t} )</th>
<th>(6) ( \lambda_{e,t} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium</td>
<td>-0.09*</td>
<td>0.03</td>
<td>Portugal</td>
<td>-0.06</td>
<td>0.06**</td>
<td></td>
</tr>
<tr>
<td>(1880–1989)</td>
<td>(0.04)</td>
<td>(0.03)</td>
<td>(1890–1995)</td>
<td>(0.04)</td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>Finland</td>
<td>-0.17**</td>
<td>-0.01</td>
<td>Spain</td>
<td>-0.08</td>
<td>0.10*</td>
<td></td>
</tr>
<tr>
<td>(1911–1995)</td>
<td>(0.05)</td>
<td>(0.03)</td>
<td>(1901–1995)</td>
<td>(0.06)</td>
<td>(0.04)</td>
<td></td>
</tr>
<tr>
<td>France</td>
<td>-0.12</td>
<td>0.09*</td>
<td>Switzerland</td>
<td>-0.06*</td>
<td>0.05*</td>
<td></td>
</tr>
<tr>
<td>(1880–1989)</td>
<td>(0.06)</td>
<td>(0.03)</td>
<td>(1880–1995)</td>
<td>(0.03)</td>
<td>(0.02)</td>
<td></td>
</tr>
<tr>
<td>Italy</td>
<td>-0.22**</td>
<td>-0.03</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1880–1995)</td>
<td>(0.06)</td>
<td>(0.06)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^*,**,***\) indicate significance at the 10, 5, and 1 percent levels, respectively; White (1980) heteroskedasticity-consistent standard errors for the coefficient estimates are given in parentheses.
equation \( \lambda_{\Delta r} \) is insignificant. This implies that the monetary fundamentals are weakly exogenous for these countries (see Engle et al., 1983). In other words, when deviations from the long-run equilibrium occur in Belgium, Finland, and Italy, it is primarily the exchange rate that adjusts to restore long-run equilibrium over our sample, rather than the monetary fundamentals. For Portugal and Spain, the results are reversed: \( \lambda_{\Delta r} \) is insignificant, while \( \lambda_{\Delta e} \) is significant, so that the exchange rate is weakly exogenous for these countries over our sample. When deviations from the long-run equilibrium occur in Portugal and Spain, it is the monetary fundamentals that bear the brunt of adjustment over our sample.\(^{28}\)

For France and Switzerland, an intermediate result obtains, as both error-correction coefficients, \( \lambda_{\Delta e} \) and \( \lambda_{\Delta r} \), are significant (and have the correct sign), so that neither the exchange rate nor the monetary fundamentals are weakly exogenous. Both the nominal exchange rate and the monetary fundamentals adjust to restore long-run equilibrium for these two countries over our sample. The different adjustment mechanisms at work in the different countries over the last century likely reflect varying degrees of commitment to nominal exchange rate stability.\(^{29}\)

5. Nominal exchange rate forecasting

An important strand of the extant literature investigates the forecasting performance of the monetary model of exchange rate determination. In their seminal paper, Meese and Rogoff (1983) report that out-of-sample forecasts of monetary models cannot outperform a naive random walk model for U.S. dollar exchange rates for Germany, Japan, and the United Kingdom during the 1976–1981 period.\(^{30}\) However, in a well-known paper, Mark (1995) shows that past nominal exchange rate deviations from the level predicted by the simple monetary model, \( z_t = e_t - [(m^*_t - m_t) - (y^*_t - y_t)] \), are useful in predicting U.S. dollar exchange rates at longer horizons for the period 1973–1991. The Mark (1995) finding is noteworthy, given the pessimism generated by Meese and Rogoff...

\(^{28}\)We combine the monetary fundamentals in (6) and (7) to facilitate the interpretation of the forecasting results reported in Section 5. In the working paper antecedent to the present paper, we also consider a trivariate VECM that separates out the monetary fundamentals into the relative money supply and relative income level. We find no country for which the error-correction coefficient on the relative income level is significant, so it is the relative money supply that adjusts in order to bring the monetary fundamentals in line with the exchange rate.

\(^{29}\)In the working paper antecedent to the present paper, we test the stability of the individual equations of the VECM. The nominal exchange rate equation for Portugal and the fundamentals equation for Finland and France exhibit the greatest evidence of structural change.

\(^{30}\)Surveys by Frankel and Rose (1995) and Taylor (1995) conclude that structural exchange rate models in general have not done well at explaining or predicting exchange rate movements during the modern floating exchange rate period.
(1983). In the spirit of Meese and Rogoff (1983) and Mark (1995), we examine the out-of-sample forecasting performance of the simple monetary model using our long spans of data for the countries in Table 5.

Mark (1995) computes recursive out-of-sample forecasts at the \( k \)-horizon based on monetary fundamentals. He estimates the following equation through period \( t_0 \), where \( T \) is the size of the available sample, in order to generate the first \( k \)-horizon forecast for the monetary model:

\[
\hat{e}_{t_0+k} - e_{t_0} = \hat{\alpha}(k; T_0) + \hat{\beta}(k; T_0)z_{t_0}.
\]  

Eq. (8) is then re-estimated using data through period \( t_0 + 1 \) in order to generate a second \( k \)-horizon forecast for the monetary model, and this process is continued through period \( T - k \). These \( k \)-horizon forecasts are then compared to the \( k \)-horizon forecasts from a naïve random walk model. The forecasts are compared using Theil’s \( U \), the ratio of the root mean squared prediction error (RMSE) for the monetary model to the RMSE for the random walk model, and the Diebold and Mariano (1995) test for equal predictive ability based on the MSE criterion. Mark (1995) finds that forecasts from the monetary model are often superior to those of the naïve random walk model, especially at longer horizons. Berkowitz and Giorgianni (2001) challenge the robustness of Mark’s (1995) findings by showing that they hinge critically on the assumption that \( z \) is stationary (that is, that nominal exchange rates and monetary fundamentals are cointegrated). This is problematic for Mark (1995), as he fails to find evidence of cointegration between nominal exchange rates and monetary fundamentals for his post-Bretton Woods data. However, we do find evidence of cointegration for the countries in Table 4, so the stationarity of \( z \) is much less of an issue for our data.\(^{31}\)

We follow Mark (1995) and compare forecasts from the monetary model, (8), with those obtained from a simple random walk with drift model.\(^{32}\) Recent theoretical work by McCracken (1999) and Clark and McCracken (2001) is relevant for our forecasting exercise. McCracken (1999) shows that while the popular Diebold and Mariano (1995) statistic has a standard asymptotic distribution when it is used to compare one-step-ahead forecasts between nonnested models, it has a nonstandard distribution when used to compare forecasts between two nested models. When comparing (8) against the random walk with drift model, we are, of course, comparing nested models. Clark and McCracken (2001) show that similar results hold for the Ericsson (1992) and Harvey et al. (1998) forecast encompassing tests. While there are no theoretical results for tests beyond


\(^{32}\)Actually, Mark (1995) compares forecasts from (8) with those of a random walk without drift. Kilian (1999) argues that forecasts from (8) should be compared to those of a random walk with drift, as we do.
the one-step-ahead horizon for nested models (at the time of the writing of this paper), Berben and van Dijk (1998) and Berkowitz and Giorgianni (2001) show that the one-step-ahead horizon is the most important horizon for the predictive regression (8).

We use five tests from Clark and McCracken (2001) to compare recursive out-of-sample one-step-ahead forecasts from (8) to those of a random walk with drift for the countries considered in Table 5. The first two tests, MSE-F and MSE-T, are versions of the popular Diebold and Mariano (1995) and West (1996) tests. They are used to test the null hypothesis that the MSE of the monetary model ($MSE_{MP}$) is equal to the MSE of the random walk with drift model ($MSE_{RW}$) against the alternative hypothesis that $MSE_{MP} \leq MSE_{RW}$. The other three tests are the ENC-T test of Harvey et al. (1998), the ENC-REG test of Ericsson (1992), and the ENC-NEW test developed by Clark and McCracken (2001). The null hypothesis for each of these tests is that forecasts from the random walk with drift model encompass the forecasts from (8). Forecast encompassing is based on optimally constructed composite forecasts. If the forecasts from the random walk with drift model encompass forecasts based on (8), this essentially means that forecasts from (8) provide no additional information that is valuable in forecasting exchange rates apart from the information already contained in the random walk with drift model. If we can reject the null of forecast encompassing, then forecasts from (8) provide information above and beyond the information already in forecasts from the random walk with drift model. For all five tests, the first recursive forecast is generated using the first half of the available sample. Inferences are based on the asymptotic critical values in McCracken (1999) and Clark and McCracken (2001). Clark and McCracken (2001) find that these asymptotic critical values work well in finite samples in extensive Monte Carlo simulations. They also establish the following ranking of the power of the various tests based on extensive Monte Carlo simulations:

ENC-NEW > MSE-F, ENC-T, ENC-REG > MSE-T.

The forecasting results are reported in Table 6. Column (2) gives the forecast period for each country, and column (3) reports Theil’s $U$ ($RMSE_{MP}/RMSE_{RW}$). There is considerable evidence of exchange rate predictability based on monetary fundamentals for Belgium, Italy, and Switzerland. These results are consistent with those in Table 5, where the error-correction coefficient in the exchange rate equation is significant for Belgium, Italy, and Switzerland. For these three countries, we expect the exchange rate to adjust to restore the long-run monetary equilibrium, and thus monetary fundamentals should be helpful in predicting future exchange rates. In contrast, there is no evidence that monetary fundamentals improve exchange rate forecasts for France, Portugal, and Spain. Again, this is consistent with the results in Table 5, where the error-correction coefficient in the exchange rate equation is insignificant for Portugal and Spain and only significant
Table 6
Out-of-sample one-year-ahead forecasting results

<table>
<thead>
<tr>
<th>Country</th>
<th>Forecast period</th>
<th>U*</th>
<th>MSE-F</th>
<th>MSE-T</th>
<th>ENC-NEW</th>
<th>ENC-T</th>
<th>ENC-REG</th>
</tr>
</thead>
<tbody>
<tr>
<td>Belgium (1880–1989)</td>
<td>1936–1989</td>
<td>0.98</td>
<td>2.66*</td>
<td>0.66*</td>
<td>2.27*</td>
<td>1.11</td>
<td>1.64*</td>
</tr>
<tr>
<td>Finland (1911–1995)</td>
<td>1954–1995</td>
<td>1.02</td>
<td>-1.92</td>
<td>-0.45</td>
<td>1.34</td>
<td>0.65</td>
<td>0.64</td>
</tr>
<tr>
<td>France (1880–1989)</td>
<td>1936–1989</td>
<td>1.02</td>
<td>-2.27</td>
<td>-1.04</td>
<td>-0.78</td>
<td>-0.72</td>
<td>-0.96</td>
</tr>
<tr>
<td>Italy (1880–1995)</td>
<td>1939–1995</td>
<td>0.94</td>
<td>8.06**</td>
<td>0.69*</td>
<td>10.34**</td>
<td>1.49*</td>
<td>2.90**</td>
</tr>
<tr>
<td>Portugal (1890–1995)</td>
<td>1944–1995</td>
<td>1.01</td>
<td>-0.62</td>
<td>-0.29</td>
<td>-0.08</td>
<td>-0.07</td>
<td>-0.11</td>
</tr>
<tr>
<td>Spain (1901–1995)</td>
<td>1949–1995</td>
<td>1.03</td>
<td>-2.82</td>
<td>-0.67</td>
<td>0.20</td>
<td>0.09</td>
<td>0.11</td>
</tr>
<tr>
<td>Switzerland (1880–1995)</td>
<td>1939–1995</td>
<td>0.99</td>
<td>1.53*</td>
<td>1.15*</td>
<td>0.96</td>
<td>1.43*</td>
<td>1.55*</td>
</tr>
</tbody>
</table>

* ** indicates significance at the 10, 5, and 1 percent levels, respectively.

U is the ratio RMSE_{MF}/RMSE_{RW}, where RMSE_{MF} is the root mean square error for the forecasting model with the monetary fundamentals, \( \Delta r_t = \alpha + \beta_r (r_{t-1} - f_{,t-1}) \), where \( f_r = (m_{,t} - m_{,r}) - (y_{,t} - y_{,r}) \), and RMSE_{RW} is the root mean square error for the forecasting model without the monetary fundamentals, \( \Delta r_t = \alpha \); for Finland and Portugal, \( f_r = (m_{,t} - m_{,r}) \); the initial recursive forecasts use the first half of the sample.

One-sided (upper-tail) test of \( H_0: MSE_{RW} = MSE_{MF} \) vs. \( H_1: MSE_{RW} > MSE_{MF} \); McCracken (1999) 10, 5, and 1 percent critical values equal 0.751, 1.548, and 3.584, respectively.

One-sided (upper-tail) test of \( H_0: MSE_{RW} = MSE_{MF} \) vs. \( H_1: MSE_{RW} > MSE_{MF} \); McCracken (1999) 10, 5, and 1 percent critical values equal 0.443, 0.771, and 1.436, respectively.

One-sided (upper-tail) test of \( H_0: \) Forecasts for the model without the monetary fundamentals encompass the forecasts for the model with the monetary fundamentals; Clark and McCracken (2001) 10, 5, and 1 percent critical values equal 0.984, 1.584, and 3.209, respectively.

One-sided (upper-tail) test of \( H_0: \) Forecasts for the model without the monetary fundamentals encompass the forecasts for the model with the monetary fundamentals; Clark and McCracken (2001) 10, 5, and 1 percent critical values equal 0.955, 1.331, and 2.052, respectively.
at the 10 percent level for France. For these three countries, the error-correction coefficient in the fundamentals equation is significant, indicating that it is primarily the monetary fundamentals—instead of the nominal exchange rate—that adjust to restore the long-run monetary equilibrium. Given that the monetary fundamentals do the adjusting, it is not surprising to find that nominal exchange rate deviations from the long-run equilibrium are not helpful for predicting future exchange rates for France, Portugal, and Spain. There is relatively little evidence that monetary fundamentals improve forecasts for Finland, although the ENC-NEW test, which Clark and McCracken (2001) find to be the most powerful of the five tests, is significant at the 10 percent level.\textsuperscript{33}

The results in Tables 5 and 6 have important implications for tests of the monetary model based on exchange rate predictability. Berkowitz and Giorgianni (2001) recommend testing for cointegration and estimating a VECM before proceeding to exchange rate forecasting, as the forecast results will depend critically on the existence of cointegration and weak exogeneity. Our results strongly support their recommendation. If we only look at the forecasting results in Table 6, we would conclude that the monetary model does not hold for three or four of the countries in Table 6, as the monetary fundamentals do not improve exchange rate forecasts for these countries. However, the results in Tables 2–4 indicate that there is support for the long-run monetary model for all of the countries in Table 6. As discussed above, the discrepancy can be explained by the results in Table 5: the inability of monetary fundamentals to improve exchange rate forecasts in some countries can largely be attributed to the weak exogeneity of the exchange rate in those countries.

6. Conclusion

Groen (2000) and Mark and Sul (2001) test the monetary model using panel data from the modern float, motivated by studies that find support for long-run PPP using panel data from the modern float. Similarly, we test the monetary model using data spanning the late nineteenth or early twentieth century to the late twentieth century, motivated by studies that find support for long-run PPP using long spans of data. Using unit root and cointegration tests, we find considerable support for a simple form of the long-run monetary model of U.S. dollar exchange rate determination for France, Italy, Spain, and the Netherlands. We find moderate

\textsuperscript{33}Given that Mark (1995) examines forecasts at multiple horizons during the modern float, in the working paper antecedent to the present paper we use Theil’s $U$ to compare out-of-sample forecasts for the monetary model with those of the random walk with drift model at horizons of one to five years over the post-1972 period. We base inferences on the Kilian (1999) bootstrap procedure, as Kilian (1999) shows that this improves on the bootstrap procedure in Mark (1995). The results are generally similar to those in Table 6.
support for Belgium, Finland, and Portugal and weaker support for Switzerland. Together with Groen (2000) and Mark and Sul (2001), we show that support for the long-run monetary model of exchange rate determination is not as elusive as it once appeared. However, our results also suggest that the support for the monetary model in Groen (2000) and Mark and Sul (2001) may be overstated. We identify a number of countries—Australia, Canada, Denmark, Norway, Sweden, and the United Kingdom—for which the long-run monetary model does not hold, while the panel cointegration tests in Groen (2000) and Mark and Sul (2001) require one to accept the monetary model for each member of the entire panel. It would be useful to examine the robustness of the Groen (2000) and Mark and Sul (2001) results to various subpanels and to formally test for heterogeneity across panel members.

For the countries for which we find support for the simple long-run monetary model, we consider two additional topics. First, we estimate vector error-correction models for nominal exchange rates and monetary fundamentals in order to test for weak exogeneity. This analysis provides insight into adjustment process through which the long-run equilibrium relationship between exchange rates and fundamentals is restored after a shock. We find that the adjustment process can vary across countries. Second, we compare out-of-sample exchange rate forecasts from a naïve random walk model with those based on monetary fundamentals. Consistent with the recent work of Berben and van Dijk (1998) and Berkowitz and Giorgianni (2001), we find that there is a close connection between the out-of-sample forecast performance of the monetary model and the weak exogeneity test results.

Our results suggest directions for future research. Given that long-run PPP appears to hold for most countries over long time spans, the failure of the long-run monetary model for some countries using long spans of data must be due to instability in the long-run relationship between relative price levels and monetary fundamentals for those countries. It would thus be informative to search for instabilities in the long-run relative price level–monetary fundamentals relationship in the countries for which the long-run monetary model fails. In countries for which there is support for the long-run model, it would be interesting to examine the adjustment process to the long-run equilibrium relationship implied by the long-run monetary model in more detail by calculating impulse responses for nominal exchange rates and monetary fundamentals in a VECM framework. Finally, recent research by Taylor and Peel (2000) suggests that nominal exchange rate deviations from underlying monetary fundamentals display nonlinear mean-reversion. It would be interesting to explore this possibility for the countries for which we find support for the long-run monetary model using long spans of data.

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References


Further reading