Probability Calculus 2019/2020
Problem set 13

1. We roll a symmetric die 200 times. What is the probability that the total number of points obtained will fall into the interval $(700, 750)$?
2. We toss a symmetric coin until we obtain 1000 heads. What is the approximate probability that we will need to toss more than 2100 times?
3. An ATM operates 100 transactions daily. The amount of cash withdrawn in a single operation is a random variable with mean 300 and a standard deviation of 150. We assume different withdrawals are independent. Find an interval (narrowest possible), such that the total amount withdrawn on a given day will fall into this interval with probability at least 0.95.
4. One day, 900 men and 900 women enter a shop. Each person decides to buy (or not) an advertised box of chocolates, which is on sale, with probability $\frac{1}{2}$. What is the (approximate) probability that the number of women who make this purchase will not differ from the number of men who buy the box by more than 50?
5. How many (independent) respondents should be interrogated in order to be able to determine the percentage of non-smokers in a population with precision of at least 0.1, with probability at least 0.9, if we know from previous research that this fraction does not exceed 25%? What would happen if we did not have any a priori knowledge?
6. There are 200 seats in cinema X, and 250 seats in cinema Y. On a given Friday evening 400 individuals go to the movies: each person, independently form others, chooses one of the cinemas with probability $\frac{1}{2}$. Approximate the probability that everyone will get a seat in the cinema of his choice.

Some additional simple problems you should be able to solve on your own:

Theory (you should know going into this class)

Formulate the Central Limit Theorem and the de Moivre-Laplace Theorem

Problems (you should know how to solve after this class)

1. From a box with 5 balls numbered from 1 to 5, we draw a ball 500 times with replacement. Approximate the probability that we will draw the ball with the number "1" less than 80 times.
2. We roll a die until the sum of points obtained exceeds 350. Approximate the probability that we will roll more than 120 times.
3. We taka a sum of 10000 numbers, each of which is rounded up to 10^{-3}. Let us assume that the resulting errors are random variables from a uniform distribution over the interval $(-10^{-3}/2, 10^{-3}/2)$. Find an interval (the narrowest possible), into which the total aggregate error will fall with probability equal to at least 0.95.
4. We have a coin with an unknown probability of obtaining tails, equal to p. How many times should an experiment of tossing the coin be repeated, in order to assure that with probability of at least 0.95, the empirical fraction of tails will not differ from p by more than 0.05?
5. On a given hour, 200 clients enter McDonald's. Each one, independently of others, buys either a hamburger (with probability $\frac{1}{2}$), French fries (with probability $\frac{1}{3}$), or does not buy anything and just uses the toilet (with probability $\frac{1}{6}$). Approximate the probability that the number of guests who do not buy anything exceeds 40.