

Probability Calculus 2019/2020
Problem set 1

1. Events A and B are equally probable, and $\mathbb{P}(A) = 2\mathbb{P}(A \cap B)$. Prove that $\mathbb{P}(B) \leq \frac{2}{3}$.
2. From a deck of 52 cards we randomly draw 7 cards (without replacement). What is the probability, that
 - a) exactly 6 cards of clubs were drawn,
 - b) at least 6 cards of clubs were drawn,
 - c) no kings were drawn,
 - d) no hearts were drawn or no aces were drawn,
 - e) the first card was a heart?
3. We throw a cubic die three times. What is the probability that the same result will be obtained exactly twice?
4. 52 cards were distributed among four players, 13 each. What is the probability that each player obtained at least one card in clubs?
5. The letters A, A, L, M, M, M, S were randomly set in a series. What is the probability that the word "MAMMALS" was obtained?
6. We draw 10 numbers, with replacement, from the set $\{1, 2, \dots, 15\}$. What is the probability that the maximum value obtained is 11?
7. Paradox of Chevalier de Méré. What is more probable: that at least one six would turn up in four rolls of a die, or that at least one pair of sixes would turn up in 24 rolls of a pair of dice?
8. From the set $\{0, 1, 2, \dots, 9\}$ we draw in sequence, without replacement, four numbers: a, b, c, d . What is the probability that
 - a) a is the largest among the four?
 - b) the series (a, b, c, d) is decreasing?

Some additional problems

Theory (you should know after the first lecture and before this class):

1. Specify the axiomatic definition of probability.
2. Define the complementary event.
3. What is classical probability?

Problems (you should know how to solve after this class):

4. Events A and B are such that $\mathbb{P}(A) = \frac{1}{2}$, $\mathbb{P}(B) = \frac{2}{3}$, $\mathbb{P}(A \setminus B) = \frac{1}{3}$. Calculate $\mathbb{P}(B \setminus A)$.
5. A cubic die was rolled twice. What is more probable: obtaining a sum of points equal to 7 or rolling the same number twice?
6. A coin was tossed five times. What is the probability that heads appeared at least twice?
7. The numbers 1, 2, \dots , 7 were rearranged in a random sequence. What is the probability that no odd numbers are next to each other?