Probability Calculus Midterm Test. December 8th, 2017. Version A

Please choose 5 out of the 7 problems below and solve each one on a separate piece of paper. Please sign each paper with your name and student's number and note the version. Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. There are 6 white, 7 black and 3 yellow balls in a box. We draw a ball 5 times with replacement.
a) Calculate the probability that there is at least one white ball among the balls drawn.
b) Calculate the probability that there is at least one ball in each color among the balls drawn.
c) Calculate the expected value of the number of black balls drawn.

2. The number of road accidents in city X on a given day has a Poisson distribution with parameter 1, if it is Saturday or Sunday, and a Poisson distribution with parameter 5 if it is Monday, Tuesday, Wednesday, Thursday or Friday. We know that during two randomly chosen consecutive days there was no accident. What is the probability that there will also be no accident during two consecutive days a week later (we look at the same days of the week)?

3. A stockholder decides to invest in shares of 20 companies: S_1, S_2, ..., S_{20}. To do so, he tosses a coin (for which the probability of heads amounts to 1/5). If heads appear, he buys 50 shares of a randomly chosen company (each has the same chance of being chosen). If tails appear, he randomly chooses five companies (each choice has the same probability) and buys 10 shares of each.
a) Calculate the probability that the stockholder bought at least 10 shares of company S_1.
b) One day, the investor lost his coin and started using another one, where the probability of heads is p. He noticed that during 100 subsequent operations: he bought exactly 10 shares of company S_1 5 times, he bought 50 shares of company S_1 4 times, and in the remaining cases he did not buy any shares of company S_1. For which value of p does the empirical distribution connected with the observed sample coincide with the theoretical distribution of the number of S_1 shares bought?

4. Let X be a random variable with a CDF equal to

$$F(t) = \begin{cases} 0 & \text{if } t < -3, \\ \frac{1}{6} & \text{if } -3 \leq t < -1, \\ \frac{1}{3}t + \frac{1}{2} & \text{if } -1 \leq t < 1, \\ 1 & \text{if } t \geq 1. \end{cases}$$

a) Is X continuous? Is X discrete? Justify your answer.
b) Find the value of the CDF of random variable $Y = X^2$ at point 1.
c) Calculate the mean and the variance of random variable $Z = X + 3$ (*Hint: find the CDF of Z*).

5. Let X be a random variable with density $g(x) = \frac{8}{x^3}\mathbb{1}_{[2,\infty)}(x)$.
a) Find the quantile of rank 1/3 of X and the expected value of $4/X$.
b) Find the CDF of $Y = \sqrt{2X^2 + 1}$ and determine whether Y has a density.

6. The duration of a telephone call (in seconds) has a uniform distribution over the interval $[0, 200]$. With probability 1/20, the telephone call is a roaming connection and a proportional charge of 2 cents per second is added; in other cases the telephone cost is included in a monthly standing charge of \$50. There were 120 independent connections in May.
a) Calculate the expected value of the invoice for May.
b) Using the Poisson theorem, approximate the probability that at least three calls resulting in extra charges of at least \$1 each were made.

7. We draw an additional point on one side of a regular cubic die, and then roll this die and denote by X the number of points obtained. On which side should we draw the additional point, in order for X to have the smallest variance possible?

Probability Calculus Midterm Test. December 8th, 2017. Version B

Please choose 5 out of the 7 problems below and solve each one on a separate piece of paper. Please sign each paper with your name and student's number and note the version. Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. There are 5 white, 8 black and 2 yellow balls in a box. We draw a ball 5 times with replacement.
a) Calculate the probability that there is at least one black ball among the balls drawn.
b) Calculate the probability that there is at least one ball in each color among the balls drawn.
c) Calculate the expected value of the number of white balls drawn.

2. The number of road accidents in city X on a given day has a Poisson distribution with parameter 2, if it is Saturday or Sunday, and a Poisson distribution with parameter 4 if it is Monday, Tuesday, Wednesday, Thursday or Friday. We know that during two randomly chosen consecutive days there was no accident. What is the probability that there will also be no accident during two consecutive days two weeks later (we look at the same days of the week)?

3. A stockholder decides to invest in shares of 30 companies: S_1, S_2, ..., S_{30}. To do so, he tosses a coin (for which the probability of heads amounts to $1/4$). If heads appear, he buys 60 shares of a randomly chosen company (each has the same chance of being chosen). If tails appear, he randomly chooses six companies (each choice has the same probability) and buys 10 shares of each.
a) Calculate the probability that the stockholder bought at least 10 shares of company S_2.
b) One day, the investor lost his coin and started using another one, where the probability of heads is p. He noticed that during 90 subsequent operations: he bought 60 shares of company S_2 twice, he bought exactly 10 shares of company S_2 6 times, and in the remaining cases he did not buy any shares of company S_2. For which value of p does the empirical distribution connected with the observed sample coincide with the theoretical distribution of the number of S_2 shares bought?

4. Let X be a random variable with a CDF equal to

$$F(t) = \begin{cases} 0 & \text{if } t < -4, \\ \frac{1}{10} & \text{if } -4 \leq t < -1, \\ \frac{1}{4}t + \frac{2}{3} & \text{if } -1 \leq t < 1, \\ 1 & \text{if } t \geq 1. \end{cases}$$

a) Is X continuous? Is X discrete? Justify your answer.
b) Find the value of the CDF of random variable $Y = X^2$ at point 1.
c) Calculate the mean and the variance of random variable $Z = X + 4$ (*Hint: find the CDF of Z*).

5. Let X be a random variable with density $g(x) = \frac{64}{x^5}\mathbb{1}_{[2,\infty)}(x)$.
a) Find the quantile of rank $1/4$ of X and the expected value of $1/X$.
b) Find the CDF of $Y = \sqrt{X^2 + 5}$ and determine whether Y has a density.

6. The duration of a telephone call (in seconds) has a uniform distribution over the interval $[0, 250]$. With probability $1/60$, the telephone call is a roaming connection and a proportional charge of 4 cents per second is added; in other cases the telephone cost is included in a monthly standing charge of \$40. There were 150 independent connections in June.
a) Calculate the expected value of the invoice for June.
b) Using the Poisson theorem, approximate the probability that at least two calls resulting in extra charges of at least \$2 each were made.

7. We draw an additional point on one side of a regular cubic die, and then roll this die and denote by X the number of points obtained. On which side should we draw the additional point, in order for X to have the largest variance possible?

Probability Calculus Midterm Test. December 8th, 2017. Version C

Please choose 5 out of the 7 problems below and solve each one on a separate piece of paper. Please sign each paper with your name and student's number and note the version. Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. There are 7 white, 8 black and 3 yellow balls in a box. We draw a ball 5 times with replacement.
a) Calculate the probability that there is at least one yellow ball among the balls drawn.
b) Calculate the probability that there is at least one ball in each color among the balls drawn.
c) Calculate the expected value of the number of yellow balls drawn.

2. The number of road accidents in city X on a given day has a Poisson distribution with parameter 5, if it is Saturday or Sunday, and a Poisson distribution with parameter 1 if it is Monday, Tuesday, Wednesday, Thursday or Friday. We know that during two randomly chosen consecutive days there was no accident. What is the probability that there will also be no accident during two consecutive days three weeks later (we look at the same days of the week)?

3. A stockholder decides to invest in shares of 40 companies: S_1, S_2, ..., S_{40}. To do so, he tosses a coin (for which the probability of heads amounts to 1/3). If heads appear, he buys 80 shares of a randomly chosen company (each has the same chance of being chosen). If tails appear, he randomly chooses four companies (each choice has the same probability) and buys 20 shares of each.
a) Calculate the probability that the stockholder bought at least 20 shares of company S_3.
b) One day, the investor lost his coin and started using another one, where the probability of heads is p. He noticed that during 80 subsequent operations: he bought 80 shares of company S_3 once, he bought exactly 20 shares of company S_3 4 times, and in the remaining cases he did not buy any shares of company S_3. For which value of p does the empirical distribution connected with the observed sample coincide with the theoretical distribution of the number of S_3 shares bought?

4. Let X be a random variable with a CDF equal to

$$F(t) = \begin{cases} 0 & \text{if } t < -6, \\ \frac{1}{8} & \text{if } -6 \leq t < -2, \\ \frac{1}{6}t + \frac{1}{2} & \text{if } -2 \leq t < 2, \\ 1 & \text{if } t \geq 2. \end{cases}$$

a) Is X continuous? Is X discrete? Justify your answer.
b) Find the value of the CDF of random variable $Y = X^2$ at point 4.
c) Calculate the mean and the variance of random variable $Z = X + 6$ (*Hint: find the CDF of Z*).

5. Let X be a random variable with density $g(x) = \frac{18}{x^3}\mathbb{1}_{[3,\infty)}(x)$.
a) Find the quantile of rank 1/5 of X and the expected value of $6/X$.
b) Find the CDF of $Y = \sqrt{5X^2 + 4}$ and determine whether Y has a density.

6. The duration of a telephone call (in seconds) has a uniform distribution over the interval $[0, 180]$. With probability 1/30, the telephone call is a roaming connection and a proportional charge of 5 cents per second is added; in other cases the telephone cost is included in a monthly standing charge of \$60. There were 180 independent connections in July.
a) Calculate the expected value of the invoice for July.
b) Using the Poisson theorem, approximate the probability that at least two calls resulting in extra charges of at least \$3 each were made.

7. We paint over/erase one of the points on one side of a regular cubic die, and then roll this die and denote by X the number of points obtained. On which side should we erase the point, in order for X to have the smallest variance possible?

Probability Calculus Midterm Test. December 8th, 2017. Version D

Please choose 5 out of the 7 problems below and solve each one on a separate piece of paper. Please sign each paper with your name and student's number and note the version. Each problem will be graded on a scale from 0 to 10 points. Duration: 120 minutes.

1. There are 5 white, 6 black and 4 yellow balls in a box. We draw a ball 5 times with replacement.
a) Calculate the probability that there is at least one black ball among the balls drawn.
b) Calculate the probability that there is at least one ball in each color among the balls drawn.
c) Calculate the expected value of the number of yellow balls drawn.

2. The number of road accidents in city X on a given day has a Poisson distribution with parameter 3, if it is Saturday or Sunday, and a Poisson distribution with parameter 2 if it is Monday, Tuesday, Wednesday, Thursday or Friday. We know that during two randomly chosen consecutive days there was no accident. What is the probability that there will also be no accident during two consecutive days four weeks later (we look at the same days of the week)?

3. A stockholder decides to invest in shares of 20 companies: S_1, S_2, …, S_{20}. To do so, he tosses a coin (for which the probability of heads amounts to 2/3). If heads appear, he buys 100 shares of a randomly chosen company (each has the same chance of being chosen). If tails appear, he randomly chooses five companies (each choice has the same probability) and buys 20 shares of each.
a) Calculate the probability that the stockholder bought at least 20 shares of company S_4.
b) One day, the investor lost his coin and started using another one, where the probability of heads is p. He noticed that during 100 subsequent operations: he bought 100 shares of company S_4 twice, he bought exactly 20 shares of company S_4 15 times, and in the remaining cases he did not buy any shares of company S_4. For which value of p does the empirical distribution connected with the observed sample coincide with the theoretical distribution of the number of S_4 shares bought?

4. Let X be a random variable with a CDF equal to

$$F(t) = \begin{cases} 0 & \text{if } t < -5, \\ \frac{1}{20} & \text{if } -5 \leq t < -2, \\ \frac{1}{5}t + \frac{1}{2} & \text{if } -2 \leq t < 2, \\ 1 & \text{if } t \geq 2. \end{cases}$$

a) Is X continuous? Is X discrete? Justify your answer.
b) Find the value of the CDF of random variable $Y = X^2$ at point 4.
c) Calculate the mean and the variance of random variable $Z = X + 5$ (*Hint: find the CDF of Z*).

5. Let X be a random variable with density $g(x) = \frac{24}{x^4}\mathbb{1}_{[2,\infty)}(x)$.
a) Find the quantile of rank 2/3 of X and the expected value of $3/X$.
b) Find the CDF of $Y = \sqrt{3X^2 + 4}$ and determine whether Y has a density.

6. The duration of a telephone call (in seconds) has a uniform distribution over the interval $[0, 300]$. With probability 1/40, the telephone call is a roaming connection and a proportional charge of 4 cents per second is added; in other cases the telephone cost is included in a monthly standing charge of \$30. There were 120 independent connections in February.
a) Calculate the expected value of the invoice for February.
b) Using the Poisson theorem, approximate the probability that at least three calls resulting in extra charges of at least \$4 each were made.

7. We paint over/erase one of the points on one side of a regular cubic die, and then roll this die and denote by X the number of points obtained. On which side should we erase the point, in order for X to have the largest variance possible?