

Probability Calculus 2019/2020, Homework 9 (three problems)

Name and Surname Student's number

In the problems below, please use the following: as k – the sum of digits in your student's number; as m – the sum of the two largest digits in your student's number; and as n – the smallest digit in your student's number plus 1. For example, if an index number is 609999: $k = 42$, $m = 18$, $n = 1$.

Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

22. Let ξ , ζ be independent random variables, such that ξ has an exponential distribution with parameter n , and ζ has an exponential distribution with parameter m . Find the best linear approximation of variable $Y = \xi - \zeta$ by variable $X = \xi + \zeta$ (linear regression). Express the result as $aX + b$ for the appropriate values a and b .

ANSWER:

Solution:

23. There are two boxes. There are m balls numbered 1 to m in the first box, and k balls numbered 1 to k in the second box. We randomly select a box (both choices have the same probability) and then draw a ball from the box. Let X and Y denote the number of the chosen box and the number of the drawn ball, respectively. Find $\mathbb{E}(X|Y = n)$.

ANSWER:

Solution:

24. Let (X, Y) be a random vector from a distribution with density $g(x, y) = 3n^{-3}x1_{\{0 \leq y \leq x \leq n\}}$. Calculate $\mathbb{E}(kX + Y^m | X)$.

ANSWER:

Solution: