Probability Calculus 2019/2020, Homework 8 (two problems)

Name and Surname ..... Student's number .....

In the problems below, please use the following: as k ***– the sum of digits in your student's number; as*** m ***– the sum of the two largest digits in your student's number; and as*** n ***– the smallest digit in your student's number plus 1. For example, if an index number is*** 609999***:*** $k = 42$***,*** $m = 18$***,*** $n = 1$***.***
Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

20. Let (X, Y) be a random vector such that

$$\mathbb{P}((X,Y) = (0,0)) = \frac{m}{m+k}, \qquad \mathbb{P}((X,Y) = (0,n)) = \frac{n}{m+k},$$

$$\mathbb{P}((X,Y) = (n,0)) = \frac{a}{m+k}, \qquad \mathbb{P}((X,Y) = (n,n)) = \frac{k-a-n}{m+k}.$$

For which value of parameter a are the variables X and Y independent?

ANSWER:

Solution:

21. Let X, Y be independent variables such that X has a uniform distribution over the interval $[-m, n]$, and Y has an exponential distribution with parameter k. Calculate $\mathbb{E}e^{XY}$.

Hint: decompose the double integral into iterated integrals with integration with respect to y *inside.*

ANSWER:

Solution: