

Probability Calculus 2019/2020, Homework 5 (three problems)

Name and Surname ..... Student's number .....

*In the problems below, please use the following: as  $k$  – the sum of digits in your student's number; as  $m$  – the sum of the two largest digits in your student's number; and as  $n$  – the smallest digit in your student's number plus 1. For example, if an index number is 609999:  $k = 42$ ,  $m = 18$ ,  $n = 1$ .*

*Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).*

12. Find the quantile of rank  $m/k$  for variable  $X$  with a density  $g(x) = \frac{2 \ln x}{n^2 x} \mathbb{1}_{[1, e^n]}(x)$ .

Hint:  $\int \frac{\ln x}{x} dx = \frac{1}{2}(\ln x)^2$ .

ANSWER:

Solution:

13. Let  $X$  be a random variable such that  $\mathbb{P}(X = -n) = \frac{1}{m}$ ,  $\mathbb{P}(X = m) = \frac{1}{k}$ ,  $\mathbb{P}(X = k) = 1 - \frac{1}{m} - \frac{1}{k}$ . Calculate  $\mathbb{E}(kX - mn)$ .

ANSWER:

Solution:

14. Assume that we might participate in a game, after paying a charge. The game is as follows: we do two rolls of a cubic die, for which the probability of obtaining a six amounts to  $m/k$  and the remaining numbers appear with equal probabilities. For each six that we roll we will receive  $kn$  dollars; additionally, if a pair of ones, a pair of twos, a pair of threes, a pair of fours or a pair of fives appear, we will receive  $mn$  dollars. If anything else appears, we do not win or lose anything. What should the charge be equal to, in order to make participation in the game profitable for the player?

*The solution should be in the form of an inequality "game charge < ...".*

ANSWER:

charge <

Solution: