Probability Calculus 2019/2020, Homework 5 (three problems)

Name and Surname ..... Student's number .....

In the problems below, please use the following: as k ***– the sum of digits in your student's number; as*** m ***– the sum of the two largest digits in your student's number; and as*** n ***– the smallest digit in your student's number plus 1. For example, if an index number is*** 609999***:*** $k = 42$***,*** $m = 18$***,*** $n = 1$***.***
Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

12. Find the quantile of rank m/k for variable X with a density $g(x) = \dfrac{2\ln x}{n^2 x}\mathbb{1}_{[1,e^n]}(x)$.

Hint: $\displaystyle\int \frac{\ln x}{x}\mathrm{d}x = \frac{1}{2}(\ln x)^2$.

ANSWER:

Solution:

13. Let X be a random variable such that $\mathbb{P}(X = -n) = \frac{1}{m}$, $\mathbb{P}(X = m) = \frac{1}{k}$, $\mathbb{P}(X = k) = 1 - \frac{1}{m} - \frac{1}{k}$. Calculate $\mathbb{E}(kX - mn)$.

ANSWER:

Solution:

14. Assume that we might participate in a game, after paying a charge. The game is as follows: we do two rolls of a cubic die, for which the probability of obtaining a six amounts to m/k and the remaining numbers appear with equal probabilities. For each six that we roll we will receive kn dollars; additionally, if a pair of ones, a pair of twos, a pair of threes, a pair of fours or a pair of fives appear, we will receive mn dollars. If anything else appears, we do not win or lose anything. What should the charge be equal to, in order to make participation in the game profitable for the player?

The solution should be in the form of an inequality "game charge $< \ldots$".

ANSWER: charge $<$

Solution: