Probability Calculus 2019/2020, Homework 4 (three problems)

Name and Surname ..... Student's number .....

In the problems below, please use the following: as k ***– the sum of digits in your student's number; as*** m ***– the sum of the two largest digits in your student's number; and as*** n ***– the smallest digit in your student's number plus 1. For example, if an index number is*** 609999***:*** $k = 42$***,*** $m = 18$***,*** $n = 1$***.***
Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

9. We have m coins at our disposal. The chance of tossing heads for the j-th coin amounts to $\frac{n+j-1}{n+j}$, $j = 1, 2, \ldots, m$. We toss each coin once. If heads appear on the j-th coin, we get j dollars, $j = 1, 2, \ldots, m$. Let X denote the total amount of money received from all the tosses. Find $\mathbb{P}\left(X = \frac{m(m+1)}{2} - 3\right)$.

Hint: $1 + 2 + 3 + \ldots + m = \frac{m(m+1)}{2}$.

ANSWER:

Solution:

10. Let X be a random variable from an exponential distribution with parameter $n/2$. Calculate

$$\mathbb{P}\left(e^{2X} - (m+k)\, e^{X} + mk \leq 0\right).$$

ANSWER:

Solution:

11. Let $F : \mathbb{R} \to \mathbb{R}$ be a function such that

$$F(t) = \begin{cases} 0 & \text{for } t < -m, \\ \frac{(t+m)(n+1)}{(n+m)^2} & \text{for } -m \leq t < n, \\ \frac{3}{4} & \text{for } n \leq t < k, \\ nA - \frac{B}{1+t^2} & \text{for } t \geq k. \end{cases}$$

Find A and B knowing that F is the cumulative distribution function of a random variable X such that $\mathbb{P}(X = k) = \frac{1}{m}$. Please provide only the value of parameter B in the space specified below.

ANSWER: $B =$

Solution: