Probability Calculus 2019/2020, Homework 11 (three problems)

Name and Surname .... Student's number ....

In the problems below, please use the following: as k – the sum of digits in your student's number; as m – the sum of the two largest digits in your student's number; and as n – the smallest digit in your student's number plus 1. For example, if an index number is 609999: $k = 42$, $m = 18$, $n = 1$.
Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

28. Each day, Mr Jones looks at the weather forecast and if rainfall is predicted, he takes his umbrella when leaving for work. The probability that there will be rainfall on a given day amounts to n/k. The probability that a weather forecast will be correct amounts to $1 - m^{-1}$. We assume that the prevalence of rainfall (for different days) and the forecast accuracy (for different days) are independent. Using the de Moivre-Laplace theorem, approximate the probability that during $10km$ subsequent days Mr Jones will take his umbrella to work at most $10mn$ times.

The answer should be provided as $\Phi(t)$, where Φ is the CDF of the standard normal distribution. The value of t should be provided in decimal notation, rounded to four digits.

ANSWER: $\Phi\Big(\qquad\qquad \Big)$

Solution:

29. The service of a single vehicle in a highway toll booth, measured in seconds, is a random variable from a uniform distribution over the range $[m+5, m+10n+15]$. We assume that the variables corresponding to different vehicles are independent. Using the Central Limit Theorem, approximate the probability that a single toll booth will service no more than 180 vehicles in a span of an hour.

The answer should be provided as $\Phi(t)$, where Φ is the CDF of the standard normal distribution. The value of t should be provided in decimal notation, rounded to four digits.

ANSWER: $\Phi\Big(\qquad\qquad\qquad \Big)$

Solution:

30. Let X_1, X_2, ..., X_{400} be independent random variables from a distribution with density $g(x) = \frac{2n+2k+1}{2} x^{2(n+k)} \mathbb{1}_{[-1,1]}(x)$. Using the Central Limit Theorem, approximate the probability

$$\mathbb{P}\left(\frac{X_1 + X_2 + \ldots + X_{400}}{400} - \mathbb{E}X_1 < \frac{n}{5m}\right).$$

The answer should be provided on the basis of the standard normal distribution tables; it should be in decimal notation, rounded to four digits.

ANSWER:

Solution: