Probability Calculus 2019/2020, Homework 10 (three problems)

Name and Surname .... Student's number ....

In the problems below, please use the following: as k – the sum of digits in your student's number; as m – the sum of the two largest digits in your student's number; and as n – the smallest digit in your student's number plus 1. For example, if an index number is 609999: $k = 42$, $m = 18$, $n = 1$.
Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

25. We have n coins, for which the probability of tossing heads is $1/n$, m coins for which the probability of heads is $1/m$ and k coins, for which the probability of heads is $1/k$. We toss each of the $n + m + k$ coins once. Using the Chebyshev-Bienaymé inequality, assess (from below) the probability of the event $A = \{$the total amount of heads falls into the range $(1, 5)\}$.

ANSWER: $\mathbb{P}(A) \geq$

Solution:

26. Let $X_1,\ X_2,\ \ldots$ be independent random variables such that for each $j = 1,\ 2,\ \ldots,$ the variable X_j has a normal distribution with mean $k + j^{-1}$ and variance equal to m. Calculate the limit, in terms of almost sure convergence, for the sequence

$$\frac{X_1 + X_2 + \ldots + X_{3j}}{mj + n}, \qquad j = 1,\ 2,\ \ldots.$$

Hint: If $X \sim N(a, \sigma^2)$, *then* $X - a \sim N(0, \sigma^2)$.

ANSWER:

Solution:

27. A physical experiment includes observing proton collisions. The time between the j-th collision and the $j+1$-th collision is a random variable with an exponential distribution with parameter m/k, for $j = 0, 1, 2, \ldots$; we assume that all of these variables are uncorrelated. Let S_j denote the time until the j-th collision, as measured from the beginning of the experiment. Calculate the limit, in terms of convergence in probability, of the sequence $\frac{S_j}{j}$, $j = 1, 2, \ldots$.

ANSWER:

Solution: