Probability Calculus 2018/2019
Problem set 8

1. Let X be a random variable such that

$$\mathbb{P}(X = k) = \frac{2k}{n(n+1)}, \qquad k = 1, 2, \ldots, n.$$

   Calculate the expected value and the variance of variables X and $Y = 2X + 5$.

2. The probability that on a given day the share price listed on a stock exchange with continuous trading will note its first fall at a time not exceeding t is given by

$$p(t) = \begin{cases} 1 - t^{-4} & \text{for } t \geqslant 1, \\ 0 & \text{for } t < 1. \end{cases}$$

   Calculate the mean and the variance of the time during which the share price rises (until the first fall). For which p will the p-th moment of this random variable exist?

3. Let X be a random variable with density

$$g(x) = \frac{x+2}{8} 1_{(-2,2)}(x).$$

   Calculate the mean, the variance, skewness and kurtosis of X.

4. Let X be a normal random variable with mean 1 and variance 4. Calculate the mean and the variance of variable $3X^2 + 1$.

5. Let X be an exponential random variable with parameter 1. Calculate the mean and the variance of variable $Y = \max\{X, 1\}$.

6. Let X be an exponential random variable with parameter λ. Calculate the variance, skewness and kurtosis of X.

7. 10 individuals aged 25 – 50 were interrogated about their average time of commuting to work. The responses were (in minutes): 40, 20, 20, 30, 30, 10, 60, 30, 60, 90. Find the empirical distribution μ for this sample and the value of the CDF at point 50. What is the sample average?

Some additional problems

Theory(you should know going into class 8)

1. Define the variance and the p-th moment of random variable X.

2. Define the empirical distribution and the empirical distribution function connected with a sample of $X_1, X_2, \ldots, X_n$, as well as the sample mean, variance and the p-th moment.

Problems (you should know how to solve after class 8)

1. Let X be a random variable with density $g(x) = \frac{1}{9}|x|1_{[-3,3]}(x)$. Calculate the expected value and the variance of X and $Y = 3 - 5X$.

2. Let X be a random variable with uniform distribution over $[-1, 1]$. Find the variance of $2X^5 - 1$, the kurtosis of X and the second moment of $X^2 - 1$.

3. Let X be a random variable such that

$$\mathbb{P}(X = k) = 1/21, \qquad k = -10, -9, \ldots, 10.$$

Calculate the variance of X.

4. Let X be a random variable with distribution

$$\mathbb{P}(X = -2) = 1/2, \quad \mathbb{P}(X = 3) = 1/3, \quad \mathbb{P}(X = a) = 1/6.$$

Find the value a that minimizes the variance of X.

5. Data regarding the number of accidents on a crossing for the years 2000, 2001, ..., 2010 were analyzed. The yearly numbers of casualties in this period amounted to 10, 13, 7, 18, 15, 12, 20, 24, 19, 10, 21. Find the value of the CDF for the sample at point 15, the median and the first decile of the empirical distribution.