

**Probability Calculus 2018/2019**  
**Problem set 7**

1. Let  $X$  be a random variable with a Poisson distribution with parameter  $\lambda$ . Calculate  $\mathbb{E}X$ ,  $\mathbb{E}X(X-1)$ ,  $\mathbb{E}X^2$  and  $\mathbb{E}2^X$ .
2. Let  $X$  be a random variable with density  $g(x) = \frac{4}{\pi} \frac{1}{1+x^2} 1_{[0,1]}(x)$ . Calculate  $\mathbb{E}X$  and  $\mathbb{E}(3-2X)$ .
3. Let  $X$  be a random variable with density  $g(x) = \frac{1}{2} \sin x 1_{[0,\pi]}(x)$ . Calculate  $\mathbb{E}X$  and  $\mathbb{E} \cos X$ .
4. Let  $X$  be a standard normal variable. Calculate  $\mathbb{E}e^{2X}$  and  $\mathbb{E}e^{X^2/4}$ .
5. Let  $X$  be a random variable describing the weekly income of a worker from a given factory, with a cumulative distribution function of

$$F(t) = \begin{cases} 0 & \text{for } t < 200, \\ ct^2(1500-t) & \text{for } 200 \leq t < 1000, \\ 1 & \text{for } t \geq 1000, \end{cases}$$

where  $c = 2 \cdot 10^{-9}$ . Calculate the mean income of a worker. Find the median income and the first decile of the income.

6. Each edge and each diagonal of a hexagon is either colored red, blue or green. Let  $X$  denote the number of triangles with vertices in the hexagon's vertices that are colored with a single color. Calculate  $\mathbb{E}X$ .
7. We roll a die until we obtain each possible result. Find the mean number of rolls.
8. There are  $n$  students in a group. One day, the lecturer distributed graded tests randomly (he gave one test to each student). Let  $X$  denote the number of students who got their own test. Find  $\mathbb{E}X$ .

## Some additional problems

Theory (you should know coming into the seventh class):

1. The definition of the expected value of a continuous random variable  $X$ .
2. Calculation of the expected value of a transformation of a random variable.

Problems (you should know how to solve after class 7)

1. Let  $X$  be a random variable from a standard normal distribution. Calculate  $\mathbb{E}X(X + 1)$  and  $\mathbb{E}e^{3X^2/8}$ .
2. Let  $X$  be a random variable with density  $g(x) = (e - 1)^{-1}e^{1-x}1_{[0,1]}(x)$ . Find  $\mathbb{E}(X + 1)$  and  $\mathbb{E}2^{X+2}$ .
3. Let  $X$  be a random variable from a geometric distribution with parameter  $p$  ( $\mathbb{P}(X = k) = p(1 - p)^{k-1}$ ,  $k = 1, 2, \dots$ ). Find  $\mathbb{E} \min\{X, 100\}$ .
4. From the set  $\{1, 2, \dots, 49\}$  we randomly draw 6 numbers without replacement. Let  $X$  signify the number of odd numbers. Find  $\mathbb{E}X$ .
5. We have two light bulbs of type  $I$  and three light bulbs of type  $II$ . The working life of a light bulb of type  $I$  has an exponential distribution with parameter 1, and of light bulb 2 - an exponential distribution with parameter  $\frac{1}{2}$ . In case a light bulb burns out in our lamp, we change it. Let  $X$  denote the total time a lamp works until the supply of light bulbs is finished. Find  $\mathbb{E}X$ .
6. 10 girls and 10 boys are randomly paired. Calculate the expected value of the number of pairs consisting of girls only.