Probability Calculus 2018/2019
Problem set 6

1. Let X be a standard normal variable. Find the density of $Y = e^X$ and $Y = X^2$.

2. Find the quantile of rank $p = 5/16$ for a) an exponential distribution with parameter λ, b) a Binomial distribution with parameters 4 and $\frac{1}{2}$, c) a Poisson distribution with parameter 1.

3. Let X be a random variable such that

$$\mathbb{P}(X = -1) = \mathbb{P}(X = 0) = \frac{1}{4}, \quad \mathbb{P}(X = 3) = \frac{1}{3}, \quad \mathbb{P}(X = 5) = \frac{1}{6}.$$

   Calculate $\mathbb{E}X$ and $\mathbb{E}(2X - 1)$.

4. Consider the following game: we toss a symmetric coin until heads appear. If heads appear in the n-th toss, we win $(1.5)^n$ dollars. What is a reasonable price for participation in this game? And if the gain for tails in the n-th toss was 2^n dollars?

5. From a box containing n balls with numbers from 1 to n we draw a ball twice, with replacement. Let X denote the maximum number obtained. Calculate $\mathbb{E}X$.

6. The number of accidents in a city on a given day has a Poisson distribution with parameter 10 on Mondays to Fridays, and a Poisson distribution with parameter 3 on Saturdays and Sundays. Calculate the mean number of accidents in a given week.

Some additional problems

Theory (you should know coming into the sixth class):

1. What is a quantile of rank p?

2. Define the expected value of a discrete random variable X.

3. Describe the properties of the expected value operator.

Problems (you should know how to solve after class 6)

4. Let X be a random variable with values from the set $\{1, 2, \ldots, 10\}$, such that

$$\mathbb{P}(X = 1) = \frac{1}{2}, \qquad \mathbb{P}(X = 2) = \mathbb{P}(X = 3) = \ldots = \mathbb{P}(X = 10) = p.$$

Find p, $\mathbb{E}X$ and $\mathbb{E}(4X + 5)$.

5. Let X be a random variable from a distribution concentrated over the set $\{1, 2, \ldots, 10\}$, such that

$$\mathbb{P}(X = 1) = \frac{1}{2}, \qquad \mathbb{P}(X = 2) = \mathbb{P}(X = 3) = \ldots = \mathbb{P}(X = 10) = p.$$

Calculate p, $\mathbb{E}X$ and $\mathbb{E}(4X + 5)$.

6. Let X be a random variable such that $P(X = k) = \frac{1}{n}$ for $k = 2, 4, 6 \ldots, 2n$. Calculate $\mathbb{E}X$ and $\mathbb{E}(2X + 1)$.

7. Let X be a random variable from a Binomial distribution with parameters 5 and $\frac{1}{3}$. Find $\mathbb{E}X$ and $\mathbb{E}(4X - 1)$.