# Probability Calculus

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**INTRODUCTION TO RANDOM VARIABLES**

## Plan for today

Introduction to real-valued random variables

- Definition
- Examples od discrete and continuous random variables
- Definition of the distribution of a random variable
- Description of the distribution of a random variable – examples
- Cumulative Distribution Function

## Random variables – basics

1. Motivation – functions of the results of an experiment
2. Definition of a **random variable**

   *A real-valued* **random variable** *is any function* $X : \Omega \to \mathbb{R}$, *such that for all* $a \in \mathbb{R}$ *the set* $X^{-1}((-\infty, a])$ *is an event, i.e.* $X^{-1}((-\infty, a]) \in \mathcal{F}$.

   $$X^{-1}((-\infty, a]) = \{\omega \in \Omega : X(\omega) \le a\}$$

3. Examples
   - number of heads
   - sum of points on dice
   - the distance to a given point

## Random variables – distribution

4. Functions of random variables
5. Examples of descriptions of random variables.
6. Definition of a random v. **distribution**

   *The* **probability distribution of a random variable** $X$ *(real-valued) is the probability* $\mu_X$ *on* $(\mathbb{R}, \mathcal{B}(\mathbb{R}))$, *such that* $\mu_X(A) = \mathbb{P}(X \in A)$.

7. Different r.v. have the same distributions

notation: $X \sim \mu$

*we forget about* $\Omega$

## Random variables – examples

8. Examples of random variables
   - die roll
   - **discrete distributions**
   - Binomial distribution
   - Geometric distribution
   - Poisson distribution
   - uniform distribution over an interval: a **continuous distribution**
   - another continuous distribution

## Continuous random variables

# 9. Definition of a **continuous random variable** and a **density function**

*A random variable* $X$ *has a* **continuous distribution**,
*if there exists a function* $g : \mathbb{R} \to \mathbb{R}_+$,
*such that for any set* $A \in \mathcal{B}(\mathbb{R})$,
$\mu_X(A) = \mathbb{P}(X \in A) = \int_A g(x)dx.$
$g$ *is called the* **probability density function of** $X$.

# 10. The properties of density functions

- nonnegative
- normalized
- determines the distribution unequivocally

# Random variable examples – cont.

11. More examples of continuous random variables
    - uniform distribution
    - exponential distribution
    - standard normal distribution
    - general normal distribution
    - (Dirac delta)

# Random variables – the CDF

## 12. The definition of a CDF

**The Cumulative distribution function** *of a random variable* $X : \Omega \to \mathbb{R}$ *is a function* $F_X : \mathbb{R} \to [0,1]$*, such that* $F_X(t) = \mathbb{P}(X \leqslant t)$.

depends on the distribution only!
→ CDF of distribution

## 13. Examples of CDFs

- Dirac delta
- Two-point distribution – discrete distribution
- Exponential distribution