

Probability Calculus 2018/2019
Problem set 4

1. Two dice were rolled. Let X denote the sum of points on the two dice. Calculate $\mathbb{P}(X \leq 3)$, $\mathbb{P}(X = 7)$, $\mathbb{P}(X > 10.25)$, $P(X \leq 1)$.
2. A die was rolled. Let X denote the number obtained, and $Y = 7 - X$. Check that X and Y have the same distribution.
3. Let k be an integer. Let X denote the number of the Bernoulli trial – in a series of Bernoulli trials with a probability of success p – where the k -th success appeared. Determine the distribution of X .
4. There are 10 balls in a box, bearing numbers from 1 to 10. We randomly draw a ball 20 times with replacement. Let X denote the smallest number obtained. Find the distribution of X and X^2 .
5. Let X be a random variable with a uniform distribution over the interval $[0, 2]$. Find the density function and calculate $\mathbb{P}(X \in [1, 3])$.
6. Let X be a random variable from an exponential distribution with parameter $\lambda = 3$. Find
 - (a) $\mathbb{P}(X \in [3, 4])$;
 - (b) the distribution of the variable $Y = \lfloor X \rfloor$ (largest previous integer of X).
7. Let X be a random variable from an exponential distribution with parameter $\lambda > 0$, and let t and h be positive real numbers. Calculate $\mathbb{P}(X > t + h \mid X > h)$ and compare it with $\mathbb{P}(X > t)$.
8. Let X be a random variable with a density function equal to

$$g(x) = Cx^{-2}1_{[2, \infty)}(x).$$

- (a) Find C .
- (b) Calculate $\mathbb{P}(X \in [1, 12])$.

Some additional problems

Theory (you should know after the fourth lecture and before the fourth class):

1. What is a random variable? What is the distribution of the random variable?
2. Define a discrete and a continuous distribution.
3. Define the binomial, geometric, Poisson and uniform distributions.

Problems (you should know how to solve after class 4)

4. Let X be a random variable with a Poisson distribution with parameter 2. Calculate $\mathbb{P}(X = 3)$ and $\mathbb{P}(X \leq 2)$.
5. Let X denote the number of points obtained in a die roll. Find the distribution of $Y = X^2$.
6. Let X be a random variable uniformly distributed over $[-5, 8]$. Calculate $\mathbb{P}(X = -1)$ and $\mathbb{P}(X \leq 5)$.
7. Let X be a random variable with density

$$g(x) = Cx^{-3} 1_{[1,5]}(x) = \begin{cases} Cx^{-3} & \text{for } 1 \leq x \leq 5, \\ 0 & \text{otherwise.} \end{cases}$$

Find C and $\mathbb{P}\left(\frac{1}{X} \in \left[\frac{1}{2}, 3\right]\right)$.

8. Let X be a random variable from a geometric distribution with parameter p . let $k, l > 0$ be integer numbers. Calculate $\mathbb{P}(X > k + l | X > k)$ and compare with $\mathbb{P}(X > l)$.