Probability Calculus 2018/2019, Homework 9 (three problems)

Name and Surname ..... Student's number .....

In the problems below, please use the following: as k ***– the sum of digits in your student's number; as*** m ***– the sum of the two largest digits in your student's number; and as*** n ***– the smallest digit in your student's number plus 1. For example, if an index number is*** 609999***:*** $k = 42$***,*** $m = 18$***,*** $n = 1$***.***
Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

23. Let (X, Y) be a random vector such that: $\mathbb{P}((X,Y) = (0,0)) = (2k)^{-1}$, $\mathbb{P}((X,Y) = (0,1)) = m^{-1}$, $\mathbb{P}((X,Y) = (1,0)) = (2k)^{-1}$, $\mathbb{P}((X,Y) = (1,1)) = 1 - k^{-1} - m^{-1}$. Find the best linear approximation of variable Y with variable X (i.e., linear regression).

Your answer should be in the form $Y \sim aX + b$*, for appropriate coefficients* a*,* b*.*

ANSWER:

Solution:

24. We have a die, for which the probabilities of rolling 1, 2, 3, 4 points amount to $1/k$, the probability of obtaining 5 points is equal to $1/m$, and the probability of rolling a 6 amounts to $1 - 4/k - 1/m$. We roll this die until we obtain 1. Let X denote the number of rolls, and Y denote the sum of points obtained in all rolls (including the last roll). Find $\mathbb{E}(Y|X = 2)$ and $\mathbb{E}(X|Y = 5)$.

ANSWER: $\mathbb{E}(Y|X = 2) =$ $\mathbb{E}(X|Y = 5) =$

Solution:

25. Let (X, Y) be a random vector with density $g(x, y) = ne^{-nx}\mathbb{1}_{\{0 \le y \le nx\}}$. Find $\mathbb{E}(kY + m|X)$.

ANSWER:

Solution: