Probability Calculus 2018/2019, Homework 7 (three problems)

Name and Surname ..... Student's number .....

In the problems below, please use the following: as k ***– the sum of digits in your student's number; as*** m ***– the sum of the two largest digits in your student's number; and as*** n ***– the smallest digit in your student's number plus 1. For example, if an index number is*** 609999***:*** $k = 42$***,*** $m = 18$***,*** $n = 1$***.***
Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

17. $2k + 4$ individuals, all of whom had had measles, were asked about the age when they went down with the disease. The answers were: 11, 11, 12, 12, 13, 13, ..., $k + 10$, $k + 10$ (each integer number between 11 and $k + 10$ appeared twice) and a, a, a, a, where a is an integer. Find a, if we know that the empirical quantile of rank m/k for the sample obtained is equal to $m + 10$.

ANSWER:

Solution:

18. The cooling system in an energy plant relies on water from a nearby river. In order to avoid malfunction due to low water levels, the management decides to build an alternative system of cooling towers. We know that the minimum water level in year 2030 is modeled by a random variable from a distribution with density

$$g(x) = m^{-2}\left[x\mathbb{1}_{[0,m]}(x) + (2m - x)\mathbb{1}_{(m,2m]}(x)\right].$$

If the water level is above n, the plant may function without cooling towers. If the water level is between $n/2$ and n, the cooling system requires two cooling towers, and if the water level drops below $n/2$, four cooling towers must be used in order to avoid failure. Let X denote the number of cooling towers in 2030 which are required for continuous, undisturbed functioning of the energy plant. Calculate the expected value and the variance of X.

ANSWER: $\mathbb{E}X =$ $\operatorname{Var} X =$

Solution:

19. Let X be a random variable with density $g(x) = (n+2)m^{-n-2}x^{n+1}\mathbb{1}_{[0,m]}(x)$. Calculate the variance of the variable $kX + m$.

ANSWER:

Solution: