

Probability Calculus 2018/2019, Homework 6 (three problems)

Name and Surname Student's number

In the problems below, please use the following: as k – the sum of digits in your student's number; as m – the sum of the two largest digits in your student's number; and as n – the smallest digit in your student's number plus 1. For example, if an index number is 609999: $k = 42$, $m = 18$, $n = 1$.

Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

14. Let X be a random variable from a distribution with density $g(x) = C(x^n - x^m)\mathbb{1}_{[0,1]}(x)$. Calculate C and $\mathbb{E}X^k$.

ANSWER:

| | |
|----------|----------------------|
| a) $C =$ | b) $\mathbb{E}X^k =$ |
|----------|----------------------|

Solution:

15. Let X be a random variable with a CDF equal to

$$F(t) = \begin{cases} 0 & \text{if } t < 0, \\ \frac{nt}{k} & \text{if } 0 \leq t < n, \\ 1 & \text{if } t \geq n. \end{cases}$$

Find $\mathbb{E}(mX^2 + k)$.

ANSWER:

Solution:

16. We take a rectangular chessboard with $m \times k$ squares, and place a token on each square. Next, for each square we conduct a random experiment, resulting in one of the following: we remove the token (with probability $1/(n+3)$), we leave the token as it is (with probability $2/(n+3)$) or we add another token (with probability $1 - 3/(n+3)$). Find the expected value of the total number of tokens on the chessboard, after experiments for all the squares have been carried out.

ANSWER:

Solution: