Probability Calculus 2018/2019, Homework 10 (three problems)

Name and Surname .... Student's number ....

In the problems below, please use the following: as k – the sum of digits in your student's number; as m – the sum of the two largest digits in your student's number; and as n – the smallest digit in your student's number plus 1. For example, if an index number is 609999: $k = 42$, $m = 18$, $n = 1$.
Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

26. There are n while balls, $2m$ black balls and k green balls in a box. We draw a ball $n + 2m + k$ times, with replacement. Using the Chebyshev-Bienaymé inequality, provide a lower bound to the probability of event $A = \{$the number of times a black ball was drawn falls into the interval $(m, 3m)\}$.

ANSWER: $\mathbb{P}(A) \geq$

Solution:

27. Starting from January 1st, 2020, a city plans to monitor the number of car accidents. We assume that the numbers of accidents on days numbered 1, 2, ... are independently distributed random variables from a Poisson distribution with parameter kn/m. For any (non-zero) integer value j, let S_j denote the number of days between the first day and day number j (inclusive), when no accidents happened. Find the limit, in terms of almost sure convergence, of the sequence $\frac{S_j}{j}$, $j = 1, 2, \ldots$.

ANSWER:

Solution:

28. Let X_1, X_2, $\ldots$ be uncorrelated random variables such that for any $j = 1, 2, \ldots$ random variable X_j has a uniform distribution over the interval $[m - k \cdot 2^{1-j}, m]$. Find the limit, in terms of convergence in probability, of the sequence

$$\frac{X_1 + X_2 + \ldots + X_j}{j + n}, \qquad j = 1,\, 2,\, \ldots.$$

ANSWER:

Solution: