Mathematical Statistics 2018/2019, Homework 5

Name and Surname ..... Student's number .....

In the problems below, please use the following: as k – the sum of digits in your student's number; as m – the sum of the two largest digits in your student's number; and as n – the smallest digit in your student's number plus 1. For example, if an index number is* 609999*: $k = 42$, $m = 18$, $n = 1$.
Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

5. Let $X_1, X_2, \ldots, X_k$ be a random sample from a geometric distribution such that

$$P_\theta(X = x) = \theta(1-\theta)^x \text{ for } x = 0, 1, 2 \ldots,$$

where $\theta \in (0, 1)$ is an unknown parameter. Let

$$\hat{T}_c = \frac{X_1 + X_2 + \ldots X_m + c}{m}$$

be an estimator of $\frac{1}{\theta}$.

a) Find c such that the estimator $\hat{T}_c$ is unbiased.

Assuming that the true value of parameter θ is equal to $\frac{1}{n+1}$, calculate:

b) The variance of the unbiased estimator $\hat{T}_c$;

c) The Fisher information connected with a sample of size k from the specified distribution;

d) The efficiency of the unbiased estimator $\hat{T}_c$.

Hints. For this specification of the geometric distribution, the expected value is equal to $\frac{1-\theta}{\theta}$, and the variance is equal to $\frac{1-\theta}{\theta^2}$. It is not a mistake in the wording that the sample size is k, but the estimator is based on observations numbered 1 to m.

ANSWER:

a) c:	b) $\mathrm{Var}(\hat{T}_c)$:	c) $I_k(\theta)$:	d) eff $(\hat{T}_c)$:

Solution: