

## Mathematical Statistics 2018/2019, Homework 4

Name and Surname ..... Student's number .....

*In the problems below, please use the following: as  $k$  – the sum of digits in your student's number; as  $m$  – the sum of the two largest digits in your student's number; and as  $n$  – the smallest digit in your student's number plus 1. For example, if an index number is 609999:  $k = 42$ ,  $m = 18$ ,  $n = 1$ .*

*Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).*

4. Let  $X_1, X_2, \dots, X_k$  be a random sample from a distribution with density

$$f_{\theta}(x) = \frac{m}{\theta^m} x^{m-1} \mathbf{1}_{[0,\theta]}(x),$$

where  $\theta > 0$  is an unknown parameter.

Let

$$\hat{\theta} = \frac{X_1 + X_2 + \dots + X_k}{k} = \bar{X}$$

be an estimator of  $\theta$ .

a) For  $\hat{\theta}$ , calculate the bias of the estimator, assuming that the true value of parameter  $\theta$  is equal to  $n$ ;

b) For  $\hat{\theta}$ , calculate the variance of the estimator, assuming that the true value of parameter  $\theta$  is equal to  $n$ ;

c) We construct an unbiased estimator  $\hat{\theta}_U$ , based on an appropriate transformation of  $\hat{\theta}$ . Provide the value of the estimator  $\hat{\theta}_U$ , if the sample consists of observations equal to (in ascending order):

$$\frac{n+1}{k}, \quad \frac{n+2}{k}, \quad \dots, \quad \frac{n+k}{k}$$

ANSWER:

a) bias of $\hat{\theta}$ :	b) variance of $\hat{\theta}$ :	c) value of $\hat{\theta}_U$ :
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Solution: