Mathematical Statistics 2018/2019, Problem set 15
Introduction to Bayesian Statistics

1. A researcher measured the length of four insects of a newly discovered species; the measurements were (in cm): 0.38, 0.65, 0.72 and 1. The researcher assumes that the length of these insects follows a uniform distribution over the interval $(0, \theta]$, where the maximum length θ is an unknown parameter. Prior to making the measurements, the researcher believed that the maximum length of the insect is a number between $\frac{1}{2}$ and 2 (i.e., that $\theta \sim U(\frac{1}{2}, 2)$). Find the posterior distribution for the experiment. Find three estimators of the parameter θ: the Bayesian Most Probable Estimator, the Bayesian estimator for a quadratic loss function (i.e., the average of the posterior distribution) and the Bayesian estimator for a modulus loss function (i.e., the median of the posterior distribution).