Mathematical Statistics 2018/2019, Problem set 10
Hypothesis Testing

1. A new tulip species was grown. A null hypothesis that 60% of seedlings sprout is tested on the basis of the behavior of 6 seedlings.

   (a) One researcher tests the null hypothesis against the alternative that at least 60% seedlings sprout. He rejects the null if at least 5 seedlings sprout. Is the size of this test less than 0.05?

   (b) What is the size of a test performed by a different researcher, who tests the same hypotheses but rejecting the null when all seedlings sprout?

   (c) A third researcher tests the null against the alternative that the fraction of seedlings that sprout is different than 60%. Propose a critical region for this researcher for a level of significance of $\alpha = 0.06$ or less.

2. We toss a coin 10 times. We test the null hypothesis that $p = 0.5$ against the alternative that $p > 0.5$. Suppose we get 8 heads. What is the p-value of this result? Would we reject the null for a significance level of $\alpha = 0.1$? And for $\alpha = 0.05$? What would be the p-value, if we tested against the alternative that $p \neq 0.5$?

3. An experiment is successful with probability p. In order to verify a null hypothesis that $p = 0.5$, against the alternative that $p = 0.6$, we conduct 144 experiments. We reject the null hypothesis if the number of successes is more than 80. Find the significance level of the test. Find the power of the test.

4. A population may be described by a distribution with density $f(x) = \lambda e^{-\lambda x}$ for $x > 0$. We verify the null hypothesis that $\lambda = 2$ against the alternative that $\lambda = \frac{1}{4}$ with the use of a single observation: if the observation is larger than c, we reject the null hypothesis. Find c such that the significance level $\alpha = 0.01$ and find the power of the test. Would the results change if the alternate hypothesis stated that $\lambda = \frac{1}{10}$?

5. A population may be described by a distribution with density $f(x) = (\theta + 1)x^{\theta}$ for $0 < x < 1$. We test the null hypothesis that $\theta = 1$ against the alternative that $\theta = 2$. We conclude on the basis of a single observation: if the result is greater than 0.8, we reject the null. Calculate the probabilities of error of first and second kind. What is the power of the test equal to?

6. Which type of error (1st or 2nd) is the main criterion in the following cases:

   (a) A potential criminal is deemed not guilty unless is proven guilty beyond reasonable doubt;

   (b) NASA tests a component to be included in a space ship.