Mathematical Statistics 2018/2019, Problem set 9
Confidence intervals

1. Among a random group of 36 students, the average time spent daily on telephone calls was found to be 20 minutes. Assuming that the amount of time spent daily on telephone calls by a student (in minutes) is a random variable with distribution $N(\mu, 5^2)$, calculate the confidence interval for the mean duration, assuming a confidence level of 0.88. How many students should be interrogated, in order to obtain a confidence interval for the mean of length not exceeding 2, with a confidence level of 0.99?

2. A company is interested in assessing the mean daily food expenditures for a group of consumers. Assume that the expenditures follow a normal distribution, and that for a random sample of 20 persons, the mean is equal to 14.5, and the standard deviation (based on the unbiased estimator of the variance) is equal to 5.6. Calculate the realization of the 95% confidence interval for the mean. What will happen to the length of the interval if the confidence level rises?

3. A physician of world renown consulted 9 random patients. The consultations took, respectively, 24, 28, 25, 27, 22, 27, 24, 24 and 23 minutes. Assuming the duration of visits follows a normal distribution, find a 0.95 confidence level confidence interval for the mean duration of a consultation for this doctor, and a 0.9 confidence level confidence interval for the variance of the duration.

4. On the basis of a survey of 25 random typists, a confidence interval for the mean time needed to complete typing a page was assessed: $(5.588; 6.412)$. Knowing that the distribution of the time needed to type a page is normal, and the sample standard deviation was equal to 1, determine the confidence level used for the calculations.

5. Let $100\theta\%$ denote the proportion of cat owners who are more than 60 years old. In order to assess θ, a random group of 400 cat owners were interrogated. Among them, 144 were found to be older than 60. Construct a confidence interval for θ with a confidence level of 0.95.

6. How many respondents should be interrogated in order to obtain an estimate for the fraction of supporters of party X with error not exceeding 5 percentage points, with a confidence level of 0.9?

7. A group of 25 restaurants was chosen for a survey on the demand for labor. Among these restaurants, the average number o staff employed equalled 8, with a sample variance of 1.21. Build a confidence interval for the average employment level using a confidence level of 0.9, assuming the employment is distributed normally. How many more restaurants should be interviewed to obtain a confidence interval for the mean with length not exceeding 0.4?