Mathematical Statistics 2018/2019, Problem set 7

Estimator properties, part III

1. Let us assume that the number of claims from a single yearly insurance policy follows a Poisson distribution with an unknown parameter θ, and let $X_1, X_2, \ldots, X_n$ denote the number of claims from independent policies of a given insurance company. We want to estimate the probability that there will be no claims from a policy.

   (a) Let $\hat{g}_1 = \frac{1}{n}\sum_{i=1}^{n} \mathbf{1}_{X_i=0}$ be an estimator of the probability of no claims. Verify whether the estimator is unbiased and find the MSE of $\hat{g}_1$. Find the asymptotic distribution of $\hat{g}_1$ and calculate the asymptotic efficiency.

   (b) Find $\hat{g}_{MLE}$, the m.l.e. estimator of $e^{-\theta}$ (for the Poisson distribution, the probability of the variable being equal to 0 is equal to $e^{-\theta}$). Verify whether the estimator is unbiased and find the MSE (*Hint: if $X_1, X_2, \ldots, X_n$ are independent random variables from a Poisson distribution with parameter θ, then $\sum X_i \sim Poiss(n\theta)$. Hint2: $\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$*). Find the asymptotic distribution of $\hat{g}_{MLE}$ and calculate the asymptotic efficiency.

   (c) Which of the two estimators would you use and why?

2. Let us assume that the distribution of genotypes in a population is multinomial, with probabilities $\theta^2, 2\theta(1-\theta)$ and $(1-\theta)^2$. Let n_1, n_2 and n_3 denote the population numbers for the three genotypes, respectively, in a population of size n. We want to estimate θ (the probability that a single gene will be of a dominant version), and we consider three estimators $\hat{\theta}_1 = \sqrt{\frac{n_1}{n}}$, $\hat{\theta}_2 = 1 - \sqrt{\frac{n_3}{n}}$, and the maximum likelihood estimator $\hat{\theta}_{MLE}$. Find $\hat{\theta}_{MLE}$. Compare the asymptotic efficiency of the three estimators.