

Mathematical Statistics 2018 /2019, Problem set 4
Point estimation – different techniques

1. Let X denote the number of failures before the first success in a Bernoulli scheme with probability of success equal to θ , i.e. $P_\theta(X = k) = \theta(1 - \theta)^k$, for $k = 0, 1, \dots$. Find the method of moments estimator for θ . What will be the precise value of the estimator, if in a sample of n observations, the average number of failures is equal to 4?
2. Let X_1, X_2, \dots, X_n be a random sample from a distribution with density

$$f_\theta(x) = \begin{cases} \theta x^{\theta-1} & x \in (0, 1) \\ 0 & \text{otherwise} \end{cases},$$

where $\theta > 0$ is an unknown parameter.

Find the method of moments and method of quantiles estimators for θ (based on the mean and median, respectively) and the m.l.e. of θ .

Compare the values of the three estimators for a sample consisting of the following three observations:

$$\frac{1}{3}, \frac{1}{2}, \frac{5}{6}.$$

3. The size of organisms from a specific population has a distribution with density $f_\beta(x) = \frac{x}{\beta^2} e^{-\frac{x}{\beta}}$ for $x > 0$ (and 0 otherwise). A sample of n organisms is drawn. Find the m.l.e. of β , and the precise value of the estimator if the mean size in the sample is equal to 3.
4. Let X_1, X_2, \dots, X_n be a random sample from a distribution with density $f_{a,b}(x) = abx^{a-1}e^{-bx^a}$ for $x > 0$ and 0 otherwise. Find the m.l.e. of b .
5. Fish in a lake, again. (Unknown number of N fish, m caught, marked and released, n caught again, among them X marked.) Find the m.l.e. for the total number of fish.