

**Mathematical Statistics 2018 /2019, Problem set 3**  
**The Statistical Model + Distributions of Statistics**

1. There are  $n$  independent repetitions of a test. A single test ends successfully with probability  $\theta$ , such that  $\theta \in [\theta_1, \theta_2]$  is an unknown parameter, and  $\theta_1$  and  $\theta_2$  are known (and fall into the interval  $(0, 1)$ ). Describe the statistical model of the overall experiment.
2. A population of  $N$  fish lives in a lake.  $m$  of them are caught, marked and released. Some time later,  $n < m$  fish are caught, and  $X$ , the number of marked specimens among them, is determined. Describe the statistical model of the experiment.
3. A lottery box consists of 50 tickets, an unknown number of which are winning. 5 tickets are drawn one after the other, and the outcomes are noted. Describe the statistical model of the experiment. Would a change in the lottery scheme (when the drawn tickets are returned to the box) change the statistical model?
4. We observe the total number of insurance claims from a set of  $n$  independent identical contracts. Assume that the number of claims for a single contract is a Poisson variable with an unknown parameter  $\lambda$ . Describe the statistical model of the experiment.
5. A device consists of  $k$  elements of type A and  $l$  elements of type B. Assume that the duration of an element of type A follows an exponential distribution with an unknown parameter  $a$ , and the duration of an element of type B follows an exponential distribution with an unknown parameter  $b$ . The device breaks down when any of the elements stop working. We observe the life duration of the whole device. Describe the statistical model of the experiment.
6. A 6-element simple random sample is drawn from  $N(\mu, 12^2)$ . Determine the probability  $P(13, 2 < S^2 < 38, 54)$ , where  $S^2 = \frac{1}{6} \sum_{i=1}^6 (X_i - \bar{X})^2$ .
7. A typical student spends  $X$  hours weekly reading books, where  $X$  has a normal distribution with a standard deviation of 1.5. What is the probability that the standard deviation calculated on the basis of a sample of 20 random individuals will not exceed 2?