

Mathematical Statistics 2017/2018, Homework 4

Name and Surname Student's number

In the problems below, please use the following: as k – the sum of digits in your student's number; as m – the largest digit in your student's number; and as n – the smallest digit in your student's number plus 1. Please write down the solutions (transformations, substitutions etc.), and additionally provide the final answer in the space specified (the answer should be a number in decimal notation, rounded to four digits).

4. Let $X_1, X_2, \dots, X_k$ be a random sample from a distribution with density

$$f_{\theta}(x) = m\theta^m x^{-(m+1)} \mathbf{1}_{[\theta, \infty)}(x),$$

where $\theta > 0$ is an unknown parameter.

Let $\hat{\theta}_{ML}$ be the Maximum Likelihood Estimator of θ .

a) For $\hat{\theta}_{ML}$, calculate the bias of the estimator, assuming that the true value of parameter θ is equal to k ;

b) For $\hat{\theta}_{ML}$, calculate the variance of the estimator, assuming that the true value of parameter θ is equal to k ;

c) We construct an unbiased estimator $\hat{\theta}_U$, based on $\hat{\theta}_{ML}$. Provide the value of the estimator $\hat{\theta}_U$, if the sample consists of observations equal to (in ascending order):

$$\frac{1}{k+n}, \quad \frac{1}{k+n-1}, \quad \frac{1}{k+n-2}, \quad \dots, \quad \frac{1}{n+1}$$

ANSWER:

a) bias of $\hat{\theta}_{ML}$:

b) variance of $\hat{\theta}_{ML}$:

c) value of $\hat{\theta}_U$:

Solution: