

Mathematical Statistics Problem set 3
The Statistical Model + Distributions of Statistics

1. There are n independent repetitions of a test. A single test ends successfully with probability θ , such that $\theta \in [\theta_1, \theta_2]$ is an unknown parameter, and θ_1 and θ_2 are known (and fall into the interval $(0, 1)$). Describe the statistical model of the overall experiment.
2. A population of N fishes lives in a lake. m of them are caught, marked and released. Some time later, $n < m$ fish are caught, and X , the number of marked specimens among them, is determined. Describe the statistical model of the experiment.
3. A lottery box consists of 50 tickets, an unknown number of which are winning. 5 tickets are drawn one after the other, and the outcomes are noted. Describe the statistical model of the experiment. Would a change in the lottery scheme (when the drawn tickets are returned to the box) change the statistical model?
4. We observe the total number of insurance claims from a set of n independent identical contracts. Assume that the number of claims for a single contract is a Poisson variable with an unknown parameter λ . Describe the statistical model of the experiment.
5. A device consists of k elements of type A and l elements of type B. Assume that the duration of an element of type A follows an exponential distribution with an unknown parameter a , and the duration of an element of type B follows an exponential distribution with an unknown parameter b . The device breaks down when any of the elements stop working. We observe the life duration of the whole device. Describe the statistical model of the experiment.
6. A 6-element simple random sample is drawn from $N(\mu, 12^2)$. Determine the probability $P(13, 2 < S^2 < 38, 54)$, where $S^2 = \frac{1}{6} \sum_{i=1}^6 (X_i - \bar{X})^2$.
7. A typical student spends X hours weekly reading books, where X has a normal distribution with a standard deviation of 1.5. What is the probability that the standard deviation calculated on the basis of a sample of 20 random individuals will not exceed 2?