Fill in the dotted spaces [".........."]. 1 question (•) = 1 point; maximum = 16 points. Only responses in the specified places will be checked, without justifications or calculations. Fill in your responses **after having verified them**; if illegible or larded with corrections and crossings-out, the answers will be treated as wrong. You can use a simple calculator, statistical tables and one a4 sheet of paper with helpful formulas. **Communication with others is not allowed**.

NAME: .... student's number ....

1. In a a local grocery shop, the time until a client who enters makes a purchase was measured for a randomly chosen group of 10 consumers. The times were equal to

$$9, 6, 4, 11, 8, 7, 10, 9, 7, 9$$

   - The average shopping time was equal to ...., and the variance of the time (unbiased estimator) is equal to .... The skewness coefficient (formula with the median) is equal to .... and means that the distribution of shopping time is SYMMETRIC /with POSITIVE ASYMMETRY /with NEGATIVE ASYMMETRY (underline the appropriate).
   - Assuming that the shopping times are distributed normally, calculate the confidence interval for the variance of the shopping time, for a confidence level of 0.95:

....

2. Shopping times were measured for clients of grocery stores in different locations. The table below summarizes the results obtained:

Location of grocery store Duration of shopping time	Shopping Center A	Shopping Center B	Shopping Center C
Up to 8 minutes	20	30	20
More than 8 minutes	40	30	10

   - The null hypothesis that the shopping duration does not depend on store location was verified, using the chi-squared test for a significance level of 0.1. The value of the test statistic is equal to ...., the critical value of the test is equal to ...., so we REJECT /DO NOT HAVE GROUNDS TO REJECT the null hypothesis (underline the appropriate).
   - The null hypothesis that the median of shopping duration time for the studied group of 150 consumers is equal to 8 minutes was verified using a standard single-population two-sided test for the value of the fraction of purchases lasting up to 8 minutes (i.e. where $H_0 : p = \frac{1}{2}$). The empirical fraction in this case is equal to ...., the value of the appropriate test statistic is equal to ...., so for a significance level of 0.1 we REJECT /DO NOT HAVE GROUNDS TO REJECT the null hypothesis (underline the appropriate).

3. Let $X_1, X_2, \ldots, X_n$ be random variables from a distribution with density

$$f(x) = \frac{1}{3a}\mathbf{1}_{[-a,2a]}(x)$$

for an unknown parameter $a > 0$.

   - The Method of Moments estimator of a, based on the mean, is equal to

....,

and the bias of this estimator is equal to ....

- The Maximum Likelihood estimator of a depends on the MEAN /MEDIAN /MINIMUM /MAXIMUM /MODE of the sample (underline ALL the appropriate), and for a sample of $-3, 2, 4, 5$ is equal to

....

4. For a random sample of 100 consumers entering a local grocery store in district A, the average amount spent on purchases was equal to 40 PLN, with a variance (unbiased estimator) equal to 100, and for a sample of 100 consumers entering a local grocery store in district B, the average amount spent on purchases was equal to 50 PLN, with a variance (unbiased estimator) equal to 400.

   - For a significance level of 0.01, verify the null hypothesis that the average amounts spent in two districts are equal, against the alternative that in district B they are higher. The value of the test statistic is equal to ...., the critical value for the appropriate test is equal to ...., so we REJECT /DO NOT HAVE GROUNDS TO REJECT the null hypothesis (underline the appropriate).
   - A researcher knows the exact decision of the test conducted in the previous point (reject/do not reject). She wants to verify the same null hypothesis against the alternative that the average levels are different, for the same significance level, but does not want to perform the calculations again. She wants to use the results of the previous point: she can do that – THE RESULT IS GOING TO BE THE SAME /she can do that – THE RESULT IS GOING TO BE DIFFERENT /she can't use the decision from the previous point – THE RESULT MAY BE THE SAME BUT MAY ALSO BE DIFFERENT (underline the appropriate).

5. Let X be a random variable from a Binomial distribution for n trials with unknown probability of success $p \in (0, 1)$.

   - Let $\hat{p} = \frac{X+2}{n+3}$ be an estimator of p. The variance of this estimator is equal to...................., and the MSE is equal to ....
   - The Fisher information connected with the single observation X is equal to ....

6. The number of items bought by a client of a clothing store follows a Poisson distribution with an unknown parameter $\lambda > 0$. We have data regarding the shopping lists of 1000 consumers for the year 2016, when the average number of items bought was equal to 1.5 and the variance (unbiased estimator) of the number of items bought was equal to 0.8. We are interested in verifying whether the parameter λ increased with respect to 2015, when it was equal to 1.

   - Researcher A wants to verify the hypothesis of lack of changes with respect to 2015 by performing a simple test for the mean, for a significance level of 0.1. The value of the test statistic is equal to ...., the critical value of the appropriate test is equal to ...., so the decision is to REJECT /NO GROUNDS TO REJECT the null (underline the appropriate), which means that the researcher concludes that in 2016 the value of λ INCREASED /DID NOT INCREASE (underline the appropriate).
   - Researcher B verifies the same hypotheses, constructing a 90% confidence interval for the average number of items bought, and decides based on whether the confidence interval includes the value of 1 or not. The decision made by this researcher will be THE SAME /DIFFERENT /THE DECISION HAS NOTHING TO DO WITH (underline the appropriate) the decision made by the researcher A (the previous point).

7. Following the data from the previous problem, other researchers perform other calculations but with the same goal in mind. Researcher X calculates the method of moments estimator $\hat{\lambda}_X$ of the value of λ based on the variance (using the unbiased formula for the variance), and researcher Y calculates the maximum likelihood estimator $\hat{\lambda}_Y$ of the value of λ.

- The value of the estimator of researcher X, calculated on the base of the sample, is equal to ...., and the value of the estimator of researcher Y is equal to ....
- Looking at the approach of researchers X and Y, when can say that
  - The estimator λ_X is unbiased (underline the appropriate: TRUE/ FALSE)
  - The estimator λ_Y is unbiased (underline the appropriate: TRUE/ FALSE)
  - The estimator λ_Y is consistent (underline the appropriate: TRUE/ FALSE)
  - The estimator λ_Y is asymptotically normal (underline the appropriate: TRUE/ FALSE)
  - The estimator λ_Y is asymptotically efficient (underline the appropriate: TRUE/ FALSE)

8. A statistician studies the duration of trends in fashion. He assumes that the time until a new 'hot' color becomes passé, in months, follows a distribution with density

$$f(x) = \theta^2 x e^{-\theta x}$$

for $x > 0$ (and 0 otherwise), for an unknown parameter $\theta > 0$.

- Assuming that the prior distribution of θ is exponential with parameter 2, find the Bayesian Most Probable estimate of the parameter θ for a sample of n independent observations:

  ....

- The favorite color of dr. Anna Janicka is BLACK /BLUE /GREEN /GREY /ORANGE /RED /YELLOW (underline the appropriate).