Someone who rose from something has to reckon with the past.
Someone who rose from nothing has only to reckon with the present.[12]

Pier Paolo Pasolini (1968). *Pigsty*

Chapter 4. Towards a model of bequest behavior: the role of inheritances

4.1 A family tradition model of bequests

Let Y_p be the parent's income, I the inheritance received by the parent, B bequests to the child (kid), and Y_k the income of the child. Let the parent's utility U, positively depend on own consumption (C_p), on the consumption of the child (C_k), and on upholding a family tradition, viz. bequeathing less than inheriting hurts, bequeathing the same or more than what was inherited enchants. Then, the parent's utility function is given by

$$U = \alpha \ln(\underbrace{Y_p + I - B}_{C_p}) + \beta \ln(\underbrace{Y_k + B}_{C_k}) + \gamma \ln\left(\frac{B}{I}\right), \quad \alpha, \beta, \gamma > 0 . \tag{1}$$

The parent chooses the amount of bequests such as to maximize (1). Thus, for $B < Y_p + I$, the change in the parent's utility with respect to a small change in B is

$$\frac{\partial U}{\partial B} = -\frac{\alpha}{Y_p + I - B} + \frac{\beta}{Y_k + B} + \frac{\gamma}{B} . \tag{2}$$

If $B \geq Y_p + I$, the parent would not be able to consume anything at all; this case is excluded. Since U is continuously differentiable for $B \in (0, Y_p + I)$, $\lim_{B\to 0} \frac{\partial U}{\partial B} = \infty$, $\lim_{B\to (Y_p + I)} \frac{\partial U}{\partial B} = -\infty$, and

$$\frac{\partial^2 U}{\partial B^2} = -\frac{\alpha}{(Y_p + I - B)^2} - \frac{\beta}{(Y_k + B)^2} - \frac{\gamma}{B^2} < 0 \qquad \text{for } B \in (0, Y_p + I), \tag{3}$$

then (2) is strictly decreasing in $B \in (0, Y_p + I)$, and (2) "goes" from $+\infty$ to $-\infty$. Therefore, there is exactly one point where $\frac{\partial U}{\partial B} = 0$, at which, since the second derivative of U is strictly negative, the first order condition for a maximum holds. That is, there exists an optimal interior level of bequests B^*, which is uniquely determined by the first order condition

[12] Quoted from Pasolini, Pier P. (1968). "Chlew." In Pier P. Pasolini *Orgia. Chlew*. Krakow: Księgarnia Akademicka. (2003 Edition), p. 153. Translated from Polish by author after Ewa Bień's translation from Italian.

$$-\frac{\alpha}{Y_p + I - B^*} + \frac{\beta}{Y_k + B^*} + \frac{\gamma}{B^*} = 0 . \tag{4}$$

4.2 The "standard" bequests model

Let the parent's utility function be given by

$$\tilde{U} = \alpha \ln(Y_p + I - B) + \beta \ln(Y_k + B), \quad \alpha, \beta > 0 , \tag{5}$$

where Y_p is the parent's income, I the inheritance received by the parent, B bequests to the child, Y_k the income of the child, and C_p and C_k are the consumption of the parent and the consumption of the child (kid), respectively. The parent chooses the amount of bequests such as to maximize (5). Thus, for $B < Y_p + I$, we calculate the first order condition that uniquely determines the optimal level of bequests B^*, analogously to the "family tradition case"

$$\frac{\partial \tilde{U}}{\partial B} = \frac{\partial U}{\partial B} - \frac{\gamma}{B^*} = -\frac{\alpha}{Y_p + I - B^*} + \frac{\beta}{Y_k + B^*} = 0 . \tag{6}$$

4.3 Testable propositions

We are now set to state and prove a series of claims.

Claim 1. The stronger the role that adherence to family tradition plays in shaping utility (the stronger the hold of family tradition), the larger the optimal bequest.

Proof: Totally differentiating (4) with respect to B^* and γ yields

$$\frac{dB^*}{d\gamma} = -\left(B^* \left(\frac{\partial^2 U}{\partial B^2} \right) \Bigg|_{B=B^*} \right)^{-1} > 0 \quad \text{for } B \in (0, Y_p + I) , \tag{7}$$

since from (3), $\frac{\partial^2 U}{\partial B^2} < 0$. ■

Figure 4.1 illustrates Claim 1.

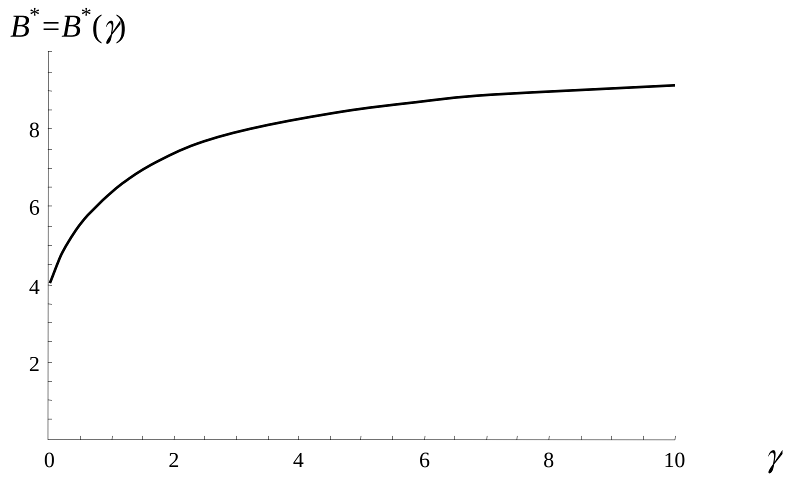

Figure 4.1: Optimal bequest $B^*(\gamma)$ as a function of γ in the family tradition model for $\alpha = \beta = 1$, $Y_p + I = 10$, $Y_k = 2$

Claim 2. The larger the inheritance, the larger the optimal bequest.

Proof: Totally differentiating (4) with respect to B^* and I yields

$$\frac{dB^*}{dI} = -\frac{\alpha}{(Y_p + I - B^*)^2}\left(\left(\frac{\partial^2 U}{\partial B^2}\right)\bigg|_{B=B^*}\right)^{-1} > 0 \quad \text{for } B \in (0, Y_p + I), \tag{8}$$

since from (3), $\frac{\partial^2 U}{\partial B^2} < 0$. ■

Figure 4.2 illustrates Claim 2.

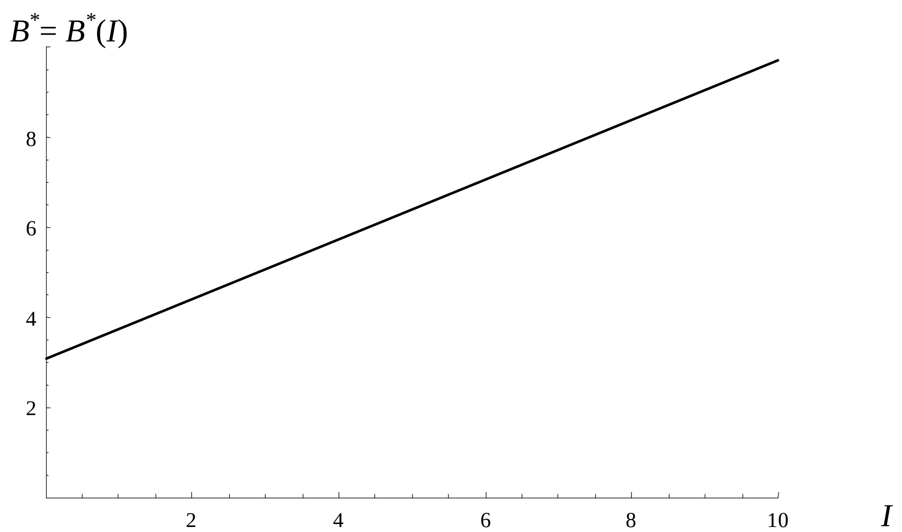

Figure 4.2: Optimal bequest $B^*(I)$ as a function of I in the family tradition model for $\alpha = \beta = \gamma = 1$, $Y_p = 5$, $Y_k = 2$

Claim 3. The positive impact of the inheritance on the optimal bequest is more pronounced in the presence of family tradition than in its absence.

Proof: We know from Claim 2 that the larger the inheritance I, the larger the optimal bequest B^*. Here we show that there is a difference in the strength of the effect of inheritance on bequests across the two models. Totally differentiating (4) with respect to B^* and I yields (8). Totally differentiating (6) with respect to B^* and I yields

$$\frac{dB^*}{dI} = -\frac{\alpha}{(Y_p + I - B^*)^2}\left(\left(\frac{\partial^2 \tilde{U}}{\partial B^2}\right)\bigg|_{B=B^*}\right)^{-1} > 0 \quad \text{for } B \in (0, Y_p + I). \tag{9}$$

Comparing (9) with (8) we find, due to

$$\frac{\partial^2 U}{\partial B^2} < \frac{\partial^2 \tilde{U}}{\partial B^2} < 0, \tag{10}$$

that $\frac{dB^*}{dI}$ is larger when family tradition plays a role in shaping utility (that is, when the parent's utility is given by (1) rather than by (5)). ■

Figure 4.3 illustrates Claim 3.

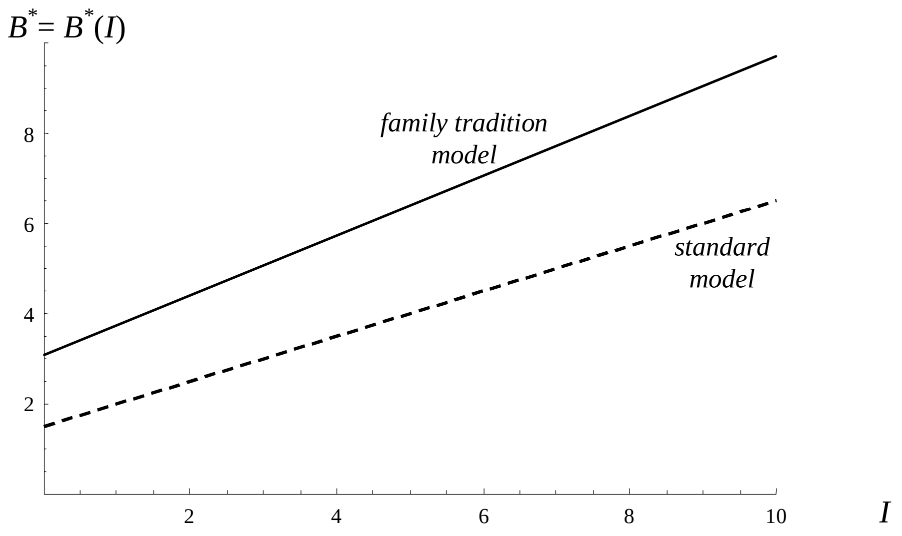

Figure 4.3: Optimal bequest $B^*(I)$ as a function of I in the family tradition and standard models for $\alpha = \beta = \gamma = 1$, $Y_p = 5$, $Y_k = 2$

Claim 4. The larger the child's income Y_k, the smaller the optimal bequest B^*.

Proof: Totally differentiating (4) with respect to B^* and Y_k yields

$$\frac{dB^*}{dY_k} = \frac{\beta}{(Y_k + B^*)^2}\left(\left(\frac{\partial^2 U}{\partial B^2}\right)\bigg|_{B=B^*}\right)^{-1} < 0 \qquad \text{for } B \in (0, Y_p + I), \tag{11}$$

since from (3), $\frac{\partial^2 U}{\partial B^2} < 0$. ■

Figure 4.4 illustrates Claim 4.

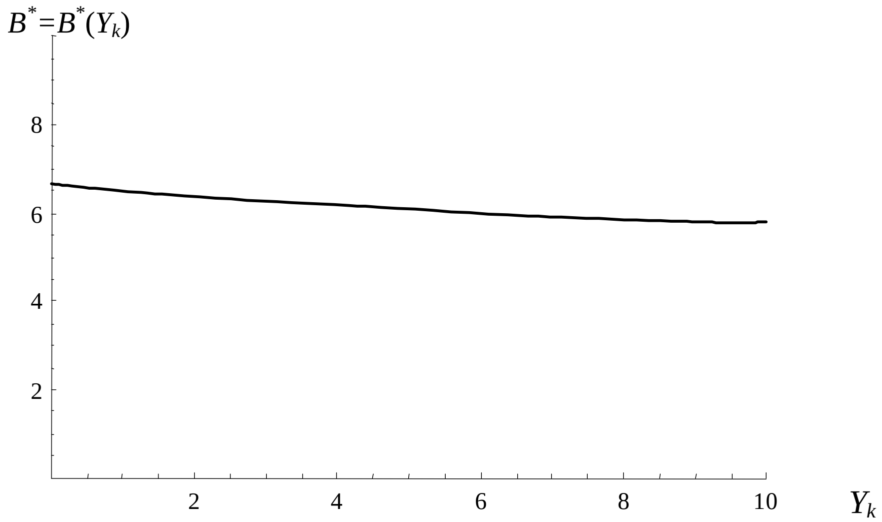

Figure 4.4: Optimal bequest $B^*(Y_k)$ as a function of Y_k in the family tradition model for $\alpha = \beta = \gamma = 1$, $Y_p + I = 10$

Claim 5. The negative impact of the child's income Y_k on the optimal bequest B^* to the child is less pronounced in the presence of the family tradition than in its absence.

Proof: We know from Claim 4 that the larger the child's income Y_k, the smaller the optimal bequest B^*. Here we show that there is a difference in the strength of this effect across the two models. Totally differentiating (4) with respect to B^* and Y_k yields (11). Totally differentiating (6) with respect to B^* and Y_k yields

$$\frac{dB^*}{dY_k} = \frac{\beta}{(Y_k + B^*)^2}\left(\left(\frac{\partial^2 \tilde{U}}{\partial B^2}\right)\bigg|_{B=B^*}\right)^{-1} < 0 \qquad \text{for } B \in (0, Y_p + I). \tag{12}$$

Comparing (12) with (11) we find, due to (10), that $\frac{dB^*}{dY_k}$ is smaller when family tradition plays a role in shaping utility (that is, when the parent's utility is given by (1) rather than by (5)). ■

Figure 4.5 illustrates Claim 5.

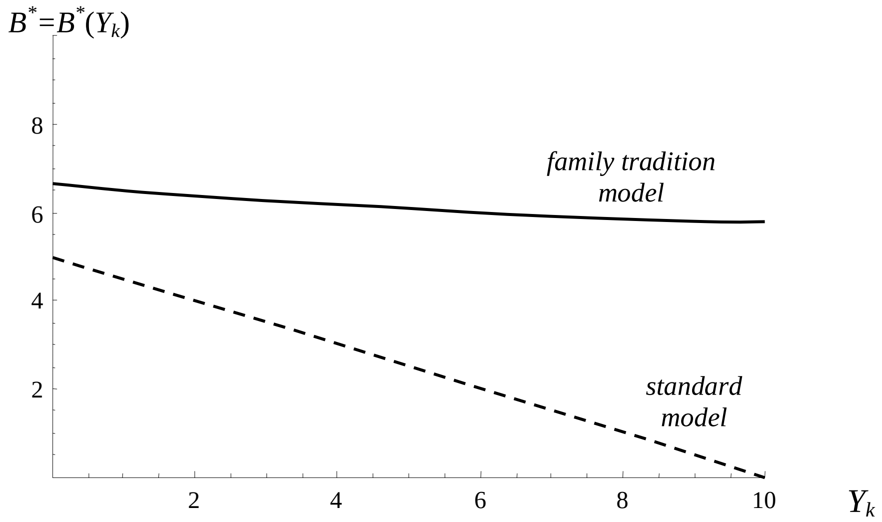

Figure 4.5: Optimal bequest $B^*(Y_k)$ as a function of Y_k in the family tradition and standard models for $\alpha = \beta = \gamma = 1$, $Y_p + I = 10$

Claim 6. The stronger the role that adherence to family tradition plays in shaping utility (the stronger the hold of family tradition), the more will the parent curtail his or her optimal consumption.

Proof: Since $C_p^* = Y_p + I - B^*$, we can express the first order condition (4) in terms of the parent's consumption,

$$-\frac{\alpha}{C_p^*} + \frac{\beta}{Y_k + B^*} + \frac{\gamma}{B^*} = 0 \,. \tag{13}$$

Totally differentiating (13) with respect to C_p^* and γ yields

$$\frac{dC_p^*}{d\gamma} = -\frac{\left.\left(\frac{\partial^2 U}{\partial B \partial \gamma}\right)\right|_{B=B^*}}{\left.\left(\frac{\partial^2 U}{\partial B \partial C_p}\right)\right|_{B=B^*}} = -\frac{C_p^{*2}}{\alpha B^*} < 0 \quad \text{for } B \in (0, Y_p + I)\,. \tag{14}$$

■

Figure 4.6 illustrates Claim 6.

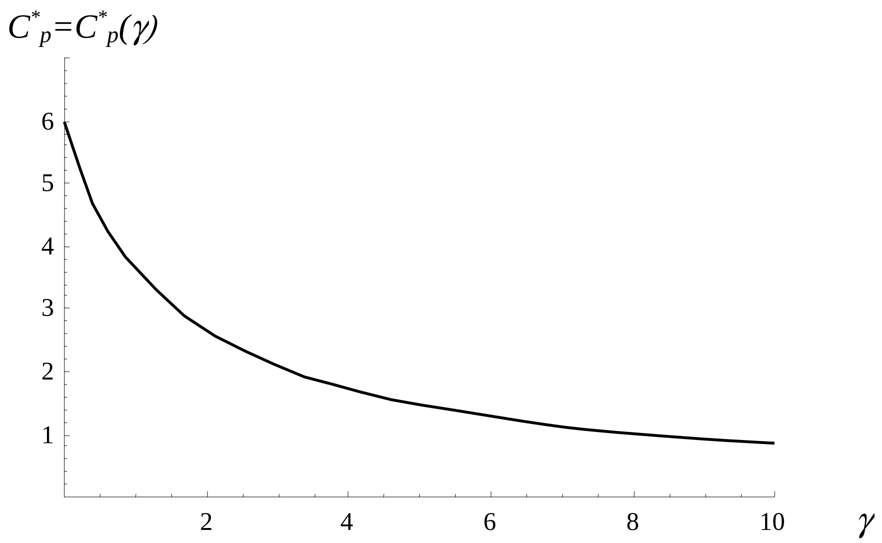

Figure 4.6: Optimal parent's consumption $C_p^*(\gamma)$ as a function of γ in the family tradition model for $\alpha = \beta = 1$, $Y_p + I = 10$, $Y_k = 2$

Claim 7. The negative impact of taxes on the optimal bequest B^* is less pronounced in the presence of family tradition than in its absence.

Proof: When bequests are taxed at rate *t*, the parent's utility function is given by

$$U = \alpha \ln(\underbrace{Y_p + I - B}_{C_p}) + \beta \ln(\underbrace{Y_k + (1-t)B}_{C_k}) + \gamma \ln\left(\frac{(1-t)B}{I}\right), \quad \alpha, \beta, \gamma > 0. \tag{1'}$$

Thus, for $B < Y_p + I$, the change in the parent's utility with respect to a small change in B is

$$\frac{\partial U}{\partial B} = -\frac{\alpha}{Y_p + I - B} + \frac{(1-t)\beta}{Y_k + (1-t)B} + \frac{\gamma}{B}. \tag{2'}$$

If $B \geq Y_p + I$, the parent would not be able to consume anything at all; this case is excluded. Since U is continuously differentiable for $B \in (0, Y_p + I)$, $\lim_{B \to 0} \frac{\partial U}{\partial B} = \infty$, $\lim_{B \to (Y_p + I)} \frac{\partial U}{\partial B} = -\infty$, and

$$\frac{\partial^2 U}{\partial B^2} = -\frac{\alpha}{(Y_p + I - B)^2} - \frac{(1-t)^2 \beta}{(Y_k + (1-t)B)^2} - \frac{\gamma}{B^2} < 0 \quad \text{for } B \in (0, Y_p + I), \tag{3'}$$

then (2') is strictly decreasing in $B \in (0, Y_p + I)$, and (2') "goes" from $+\infty$ to $-\infty$. Therefore, there is exactly one point where $\frac{\partial U}{\partial B} = 0$, at which, since the second derivative of U is strictly negative, the second order condition for a maximum holds. That is, there exists an optimal interior level of bequests B^*, which is uniquely determined by the first order condition

$$-\frac{\alpha}{Y_p + I - B^*} + \frac{(1-t)\beta}{Y_k + (1-t)B^*} + \frac{\gamma}{B^*} = 0 \,. \tag{4'}$$

Totally differentiating (4') with respect to B^* and t yields

$$\frac{dB^*}{dt} = \frac{\beta Y_k}{(Y_k + (1-t)B^*)^2}\left(\left(\frac{\partial^2 U}{\partial B^2}\right)\Bigg|_{B=B^*}\right)^{-1} < 0 \quad \text{for } B \in (0, Y_p + I)\,, \tag{15}$$

since from (3'), $\frac{\partial^2 U}{\partial B^2} < 0$. Thus, the higher the tax on bequests, the smaller the optimal bequest.

Next we analyse the difference in the strength of the tax effect between the family tradition model and the standard model. When bequests are taxed and family tradition does not count, the parent's utility function is given by

$$\tilde{U} = \alpha \ln(Y_p + I - B) + \beta \ln(Y_k + (1-t)B)\,, \quad \alpha, \beta > 0\,. \tag{5'}$$

Thus, for $B < Y_p + I$, we calculate the first order condition that uniquely determines the optimal level of bequests B^*, for the standard model of bequeathing,

$$\frac{\partial \tilde{U}}{\partial B} = \frac{\partial U}{\partial B} - \frac{\gamma}{B^*} = -\frac{\alpha}{Y_p + I - B^*} + \frac{(1-t)\beta}{(1-t)Y_k + B^*} = 0\,. \tag{6'}$$

Totally differentiating (4') with respect to B^* and t yields (15). Totally differentiating (6') with respect to B^* and t yields

$$\frac{dB^*}{dt} = \frac{\beta Y_k}{(Y_k + (1-t)B^*)^2}\left(\left(\frac{\partial^2 \tilde{U}}{\partial B^2}\right)\Bigg|_{B=B^*}\right)^{-1} < 0 \quad \text{for } B \in (0, Y_p + I)\,. \tag{16}$$

Comparing (16) with (15) we find, due to

$$\frac{\partial^2 U}{\partial B^2} < \frac{\partial^2 \tilde{U}}{\partial B^2} < 0\,, \tag{17}$$

that $\frac{dB^*}{dt}$ is larger when family tradition plays no role in shaping utility (that is, when the parent's utility is given by (5') rather than by (1')). ■

Figure 4.7 illustrates Claim 7.

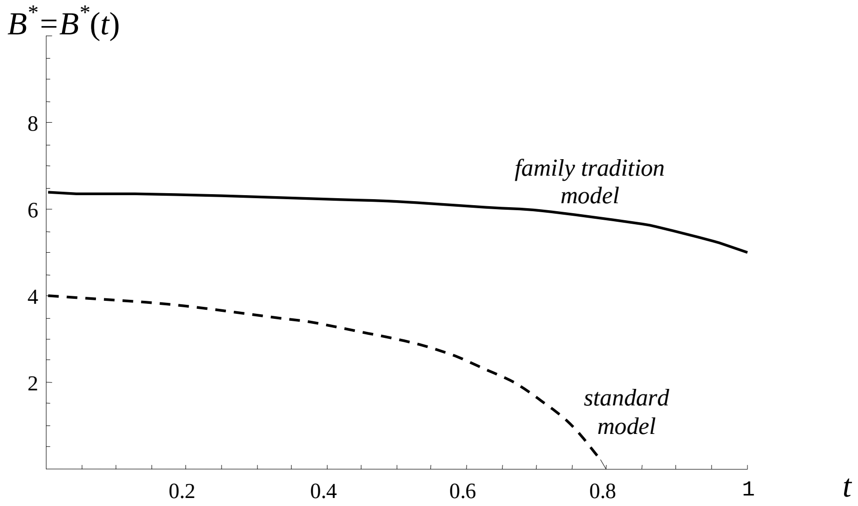

Figure 4.7: Optimal bequest $B^{*}(t)$ as a function of t in the family tradition and standard models for $\alpha = \beta = \gamma = 1$, $Y_p + I = 10$, $Y_k = 2$