

# Econometrics exam

## 2018-06-16

### EXAM RULES

1. Exam takes 90 min.
2. This exam is a closed book exam.
3. Everybody is required to sign on the list.
4. The solution of exercise should be written on the sheet on which the exercise was printed or on the additional sheets on the back of the exam.
5. All the pages with solutions should be signed. If additional sheet is used it is very important to put the number of the exercise on the top of it.
6. Only one exercise should be solved on one sheet.
7. The minimum to obtain the pass grade is to get 50% from theoretical questions and 50% from exercises.

Warsaw, 2018-06-16, \_\_\_\_\_  
*signature*

Good luck!

**Theoretical questions**

1. Derive OLS estimator for model with multiple explanatory variables.
2. Proof, that in CLRM estimator  $b$  is unbiased.

3. What test can be used to test for homoscedasticity? Give  $H_0$  and  $H_1$ . How it is connected with CLRM assumptions?
4. Show that  $s^2$  is unbiased estimator of  $\sigma^2$ ?

**Exercise 1.**

Consider the following multiple-regression model

$$y_i = \beta_1 + \beta_2 x_{2i} + \beta_3 x_{3i} + \varepsilon_i$$

where  $x_{3i} = -(2x_{2i} + 1)$

1. Using the method of Ordinary Least Squares is it possible to obtain estimates of all of the unknown parameters  $\beta_1, \beta_2, \beta_3$ ? Why or why not?
2. If not, what linear functions of these unknown parameters may be obtained? Explain

**Exercise 2.**

The median housing price was analysed. Dependent variable: logarithm median housing price was analysed (lprice). Explanatory variables: logarithm of nitrous oxide, parts per 100 mill. (lnox), logarithm of nitrous oxide, parts per 100 mill. squared (lnox2), crimes committed per capita (crime), avg number of rooms per house (rooms), avg number of rooms per house squared (rooms2), average student-teacher ratio (stratio).

| Source   | SS         | df  | MS         | Number of obs | = | 506    |
|----------|------------|-----|------------|---------------|---|--------|
| Model    | 57.2717626 | 6   | 9.54529377 | F( 6, 499)    | = | 174.41 |
| Residual | 27.3105083 | 499 | .054730478 | Prob > F      | = | 0.0000 |
|          |            |     |            | R-squared     | = | 0.6771 |
|          |            |     |            | Adj R-squared | = | 0.6732 |
|          |            |     |            | Root MSE      | = | .23395 |
| Total    | 84.5822709 | 505 | .167489645 |               |   |        |

| lprice  | Coef.     | Std. Err. | t      | P> t  | [95% Conf. Interval] |
|---------|-----------|-----------|--------|-------|----------------------|
| lnox    | 2.438683  | .9022435  | 2.70   | 0.007 | .6660183 4.211347    |
| lnox2   | -.844559  | .2597023  | -3.25  | 0.001 | -1.354804 -.3343143  |
| crime   | -.0142473 | .0013869  | -10.27 | 0.000 | -.0169722 -.0115225  |
| rooms   | -.7839091 | .1467913  | -5.34  | 0.000 | -1.072314 -.4955039  |
| rooms2  | .0799621  | .0113477  | 7.05   | 0.000 | .0576669 .1022572    |
| stratio | -.0413852 | .0057902  | -7.15  | 0.000 | -.0527614 -.030009   |
| _cons   | 10.8118   | .8937578  | 12.10  | 0.000 | 9.055804 12.56779    |

| Variable | VIF    | 1/VIF    |
|----------|--------|----------|
| lnox     | 304.70 | 0.003282 |
| lnox2    | 302.87 | 0.003302 |
| rooms    | 98.15  | 0.010189 |
| rooms2   | 97.93  | 0.010211 |
| stratio  | 1.45   | 0.689131 |
| crime    | 1.31   | 0.763569 |
| Mean VIF | 134.40 |          |

Ramsey RESET test  $F(3, 496)=7.28$ ,  $\text{Prob}>F=0.0001$

Jarque-Berra test  $\chi^2(2)=69.89$ ,  $\text{Prob}>\chi^2=0.0000$

Breusch-Pagan test  $\chi^2(1)=84.55$ ,  $\text{Prob}>\chi^2=0.0000$

1. Are the explanatory variables jointly significant?
2. Give an interpretation to coefficient of determination  $R^2$ .
3. Verify which explanatory variables are statistically significant.
4. Interpret coefficients for crime and stratio.
5. Calculate and interpret elasticity for  $\text{nox} = 10$ .
6. Calculate and interpret semi-elasticity for  $\text{rooms} = 2$ .
7. Interpret VIF test results.
8. Is the heteroscedasticity problem present in the estimated model?
9. Is the error term normally distributed in the estimated model?
10. Check whether the linear functional form of the model is valid.

11. If estimated model does not satisfy the assumptions of CLRM explain:
- (a) which assumptions of CLRM are invalid,
  - (b) what consequences does it have on statistical inference for this model,
  - (c) how the problems indicated by diagnostic tests could be solved.

**Exercise 3.**

Using a sample of 1801 individuals, the following earnings equation has been estimated:

$$\ln(\widehat{earnings}) = \underset{(0.14)}{7.06} + \underset{(0.01)}{0.15}educ + \underset{(0.01)}{0.05}experience - \underset{(0.04)}{0.21}female$$

$$R^2 = 0.17, RSS = 100$$

where the standard errors are reported in brackets.

1. Interpret the coefficient estimate on *female*.
2. Test the hypothesis that there is no difference in expected earnings between women and men. Test this hypothesis against a two-sided alternative, using the 5% significance level. You must write down: (i) the null and alternative hypotheses; (ii) the test statistic; (iii) the rejection rule.
3. Dropping *experience* and *female* from the equation gives:

$$\ln(\widehat{earnings}) = \underset{(0.18)}{6.71} + \underset{(0.01)}{0.16}educ$$

$$R^2 = 0.11, RSS = 140$$

Are experience and female jointly significant in the original equation at the 5% significance level? You must write down: (i) the null and alternative hypotheses; (ii) the test statistic; (iii) the rejection rule.

Hint:  $F_{1,1801} = 3.81$ ,  $F_{1,1797} = 3.86$ ,  $F_{2,1801} = 3.01$ ,  $F_{2,1797} = 3.51$

## Notes